Coupled data assimilation and ensemble initialization with application to multi-year ENSO prediction. (*submitted J. Climate*)

Terence O'Kane, Paul Sandery, Didier Monselesan, Pavel Sakov, Matt Chamberlain, Richard Matear, Mark Collier & Lauren Stevens

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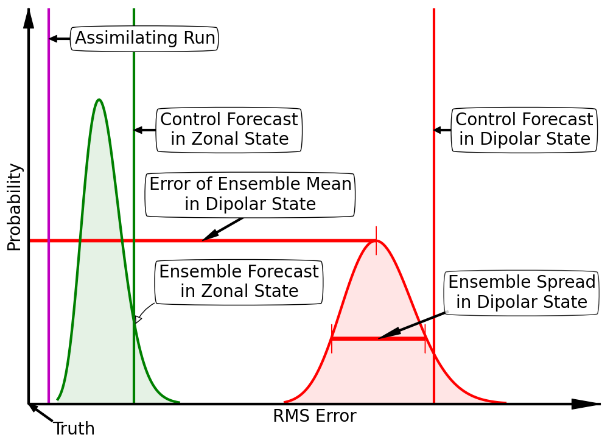
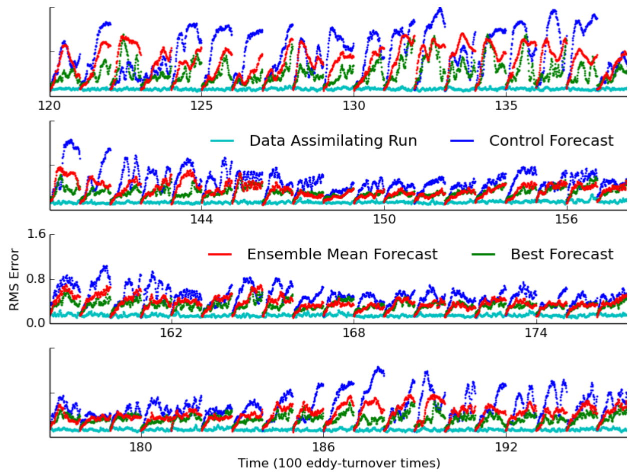
http://nespclimate.com.au/decadal-prediction/

1 Ensemble forecasting

- The aim of data assimilation is to obtain a near optimal estimate of the state of the climate system, based on observations and on short term forecasts, that provide the so-called background states with information in the data void areas.
- Ensemble prediction uses probabilistic forecasts to gain information about the predictability of the current climatic state and or the probability of the transition to a new state or regime. Larger ensemble spread is an indication of less predictability in a given regime.
- Over finite times the growth characteristics (pdf) of errors (instabilities) of the ensemble forecast can be characterised by choice of initial forecast perturbations.
- **For multi-scale systems the problem is further complicated by the presence of regimes and by the need to select spatio-temporal scales relevant to the target forecast lead time**.





2.1 Kalman filter

Consider a system with model dynamics given by Φ and considering an n-dimensional state vector $\mathbf{x}$ at timestep t, a q-dimensional state space noise process vector $\mathbf{w}$ due to disturbances and model errors and an observation vector $\mathbf{d}$ with measurement noise $\mathbf{v}$. Next, let us assume $\mathbf{v} = \hat{\mathbf{d}}$ the observation error where $\hat{\mathbf{d}}$ and $\mathbf{w}$ are white such that the analysis and forecast fields are defined as

$$\mathbf{x^{a_i}} = \langle \mathbf{x^a} \rangle + \hat{\mathbf{x}}^{\mathbf{a_i}} \tag{1a}$$

$$\mathbf{x^{f_i}} = \langle \mathbf{x^f} \rangle + \hat{\mathbf{x}}^{\mathbf{f_i}} \tag{1b}$$

$$\mathbf{d^i} = \langle \mathbf{d} \rangle + \hat{\mathbf{d}}^{\mathbf{i}} \tag{1c}$$

where $i = 1, 2, \ldots, k$ runs over the entire ensemble and where $\langle \hat{\mathbf{d}}(t)\hat{\mathbf{d}}^T(t') \rangle = \delta(t - t')\mathbf{D}(\mathbf{t}, \mathbf{t})$, $\langle \mathbf{w}(t)\mathbf{w}^T(t') \rangle = \delta(t-t')\mathbf{Q}(\mathbf{t}, \mathbf{t})$, and $\langle \mathbf{v}(t)\mathbf{w}^T(t') \rangle = 0$ for $\forall(t, t')$. The Kalman filter propagates first and second moments of $\mathbf{x}$ recursively where $\mathbf{x}(t+\delta t) = \Phi[\mathbf{x}(t), \mathbf{w}(t), t]$ and $\mathbf{d}(t) = \mathbf{H}[\mathbf{x}(t), \mathbf{v}(t), t]$ can in general be nonlinear. Φ represents the model dynamics. Assuming that the perturbation field denoted $\hat{}$ runs over the entire ensemble, we can write the recursive Kalman filter equations as

$$\mathbf{x^a} = \mathbf{x^f} + \mathbf{K}(\mathbf{d} - \mathbf{H}\mathbf{x^f}) \tag{2a}$$

$$\mathbf{P^a} = (\mathbf{I} - \mathbf{KH})\mathbf{P^f} \tag{2b}$$

$$\mathbf{K} = \mathbf{P^f}\mathbf{H^T}(\mathbf{H}\mathbf{P^f}\mathbf{H^T} + \mathbf{D})^{-1} \tag{2c}$$

where $\mathbf{K}$ is the Kalman gain, $\mathbf{P}$ is the positive definite state covariance error matrix, $\mathbf{I}$ is the indentity matrix, and $\mathbf{H}$ is the linearised observational operator mapping forecast grid point values onto observational points. A forecast model Φ maps the analysis state at time t to a forecast state at time $t + \delta t$ such that $\mathbf{x}^f(t + \delta t) = \Phi(\mathbf{x}^a(t)) + \mathbf{w}(t)$.

2.2 Kalman Filter

ETKF applies a Kalman filter with k-forecast and k-analysis perturbations

$$Z^f = \frac{1}{\sqrt{k-1}}[\mathbf{z}_1^f, \mathbf{z}_2^f, \ldots, \mathbf{z}_k^f], \tag{3}$$

$$Z^a = \frac{1}{\sqrt{k-1}}[\mathbf{z}_1^a, \mathbf{z}_2^a, \ldots, \mathbf{z}_k^a] \tag{4}$$

where the n-dimensional state vectors $\mathbf{z}_i^f = \mathbf{x}_i^f - \mathbf{x}^f$ and $\mathbf{z}_i^a = \mathbf{x}_i^a - \mathbf{x}^a$ $(i = 1, 2, \ldots, k)$ are k-ensemble forecast and analysis perturbations and $\mathbf{x}$ the ensemble mean.

For convenience, we have assumed $\mathbf{x^f}$ is the mean of k-ensemble forecasts and $\mathbf{x^a}$ is the ensemble mean analysis. The ETKF acts to choose appropriate initial forecast perturbations consistent with error covariance update equations within the vector subspace of ensemble perturbations formed as $\mathbf{P^f} = \mathbf{Z^f}\mathbf{Z^f}^{\mathbf{T}}$ and $\mathbf{P^a} = \mathbf{Z^a}\mathbf{Z^a}^{\mathbf{T}}$. In order to calculate the normal ETKF transform matrix $\mathbf{T}$, we are required to calculate the matrix of ensemble perturbations in normalised observation space, i.e.

$$\mathbf{E} = (\mathbf{D^{-1/2}HZ^f})^{\mathbf{T}}(\mathbf{D^{-1/2}HZ^f}) \tag{5}$$

where $\mathbf{H}$ maps from $\mathbf{Z}^f$ into observable space and $\mathbf{D}^{1/2}$ does the renormalization.

2.3 Kalman Filter

We are next required to find the eigenvectors $\mathbf{C}$ and eigenvalues $\mathbf{\Gamma}$ of Eqn. 5 which is equivalent to the matrix $\mathbf{Z^{f^T}HD^{-1}HZ^f}$. The transform matrix $\mathbf{T}$ is now defined in terms of the $k \times (k-1)$ matrix of non-zero eigenvalues such that

$$\mathbf{T} = \mathbf{C}(\mathbf{\Gamma} + \mathbf{I})^{\mathbf{1/2}}\mathbf{C^T} \tag{6}$$

which corresponds to the transform matrix in spherical simplex form. The ETKF transformation from forecast to analysis perturbations can be expressed as

$$\mathbf{Z}^a = \mathbf{Z}^f\mathbf{T} \tag{7}$$

where

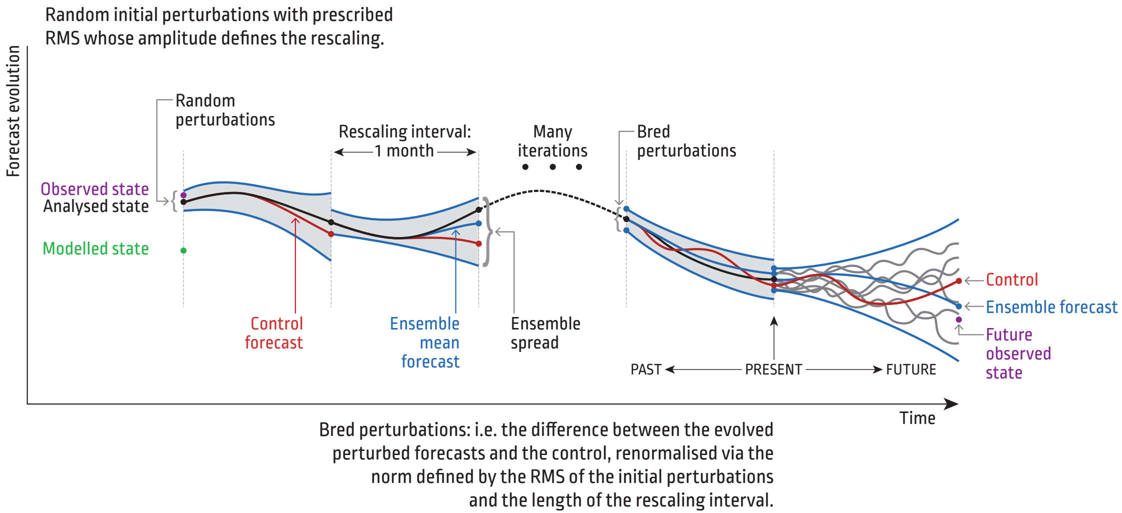
$$\mathbf{T} = \mathbf{C}(\mathbf{\Gamma} + \mathbf{I})^{-1/2} \tag{8}$$

where $\mathbf{C}$ contains the column orthonormal right SVs $(\mathbf{c}_i)$ and $\mathbf{\Gamma}$ is a diagonal matrix containing squared singular values (λ_i) of $\mathbf{R}^{-1/2}\mathbf{HZ}^f$ with $\mathbf{R}$ the $p \times p$ observational error covariance matrix and $\mathbf{H}$ the linearised observational mapping the forecast grid point values into the observational space.

BVs can be seen as replacing $\mathbf{T}$ by a renormalisation using a suitably chosen rescaling norm.

Ensemble prediction 3.1

Bred vector generation



3.2 Bred vectors

The local growth rate of the bred vectors is given by

$$g(t) = \frac{1}{\delta T} \log(\|\delta\omega(t)\| / \|\delta\omega_{t_0}\|) \tag{9}$$

where δT is the rescaling interval and $\delta\omega_{t_0}$ is the bred vector at time $t = 0$.

More generally we may define the relative amplification factor in terms of the vector of grid-point values of the bred vector of any climate variable field as $\mathbf{x}(\delta t, t)$ initiated at time t and evolved to time $\delta t + t$.

We take the L_2-norm as the *r.m.s* of the vector by $||\mathbf{x}(\delta t, t)||$ and define the amplification factor as $A(\delta t, t) = ||\mathbf{x}(\delta t, t)||/\mathbf{x}(t_0, t)||$ and the local total growth rate $\tilde{g}(t) = \frac{1}{\delta t} \log A(\delta t, t)$.

4.1 Paradigm Model

Paradigm model for ocean-tropical-extra tropical atmosphere coupling (Peña and Kalnay(2004)):

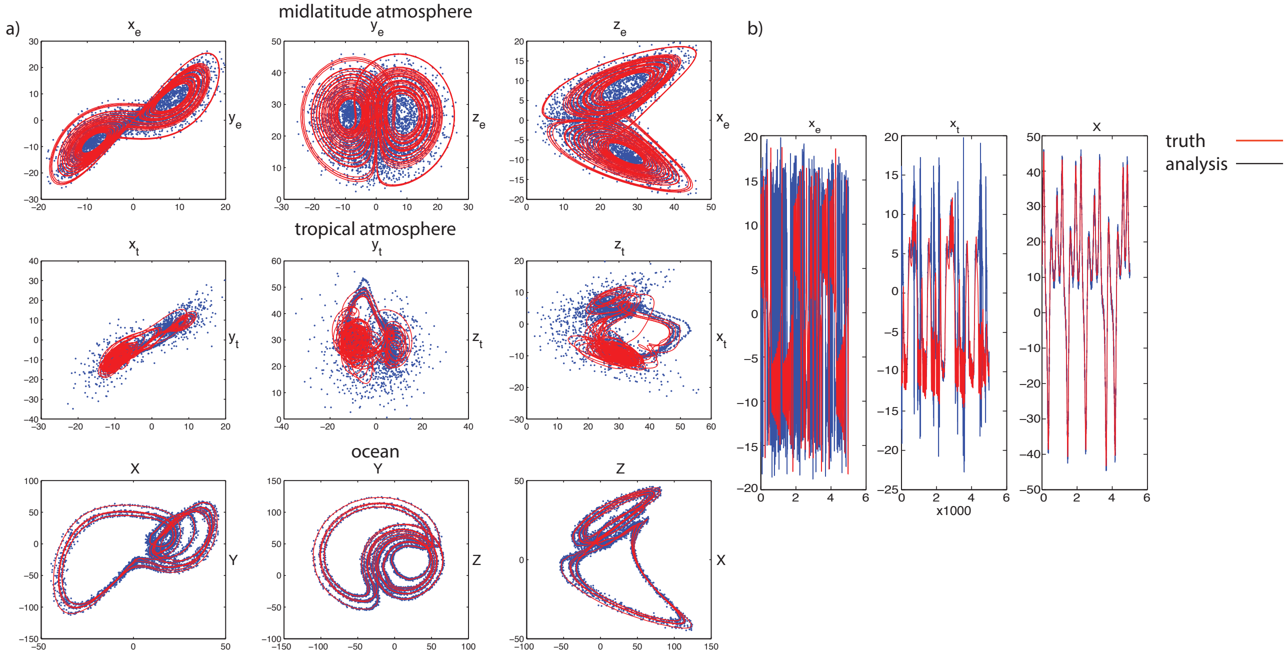
$$\dot{x}_e = \sigma(y_e - x_e) - c_e(Sx_t + k_1) \tag{10a}$$
$$\dot{y}_e = rx_e - y_e - x_e z_e + c_e(Sy_t + k_1) \tag{10b}$$
$$\dot{z}_e = x_e y_e - bz_e \tag{10c}$$

$$\dot{x}_t = \sigma(y_t - x_t) - c(SX + k_2) - c_e(Sx_e + k_1) \tag{11a}$$
$$\dot{y}_t = rx_t - y_t - x_t z_t + c(SY + k_2) + c_e(Sy_e + k_1) \tag{11b}$$
$$\dot{z}_t = x_t y_t - bz_t + c_z Z \tag{11c}$$

$$\dot{X} = \tau\sigma(Y - X) - c(x_t + k_2) \tag{12a}$$
$$\dot{Y} = \tau rX - \tau Y - \tau SXZ + c(y_t + k_2) \tag{12b}$$
$$\dot{Z} = \tau SXY - \tau bZ - c_z z_t \tag{12c}$$

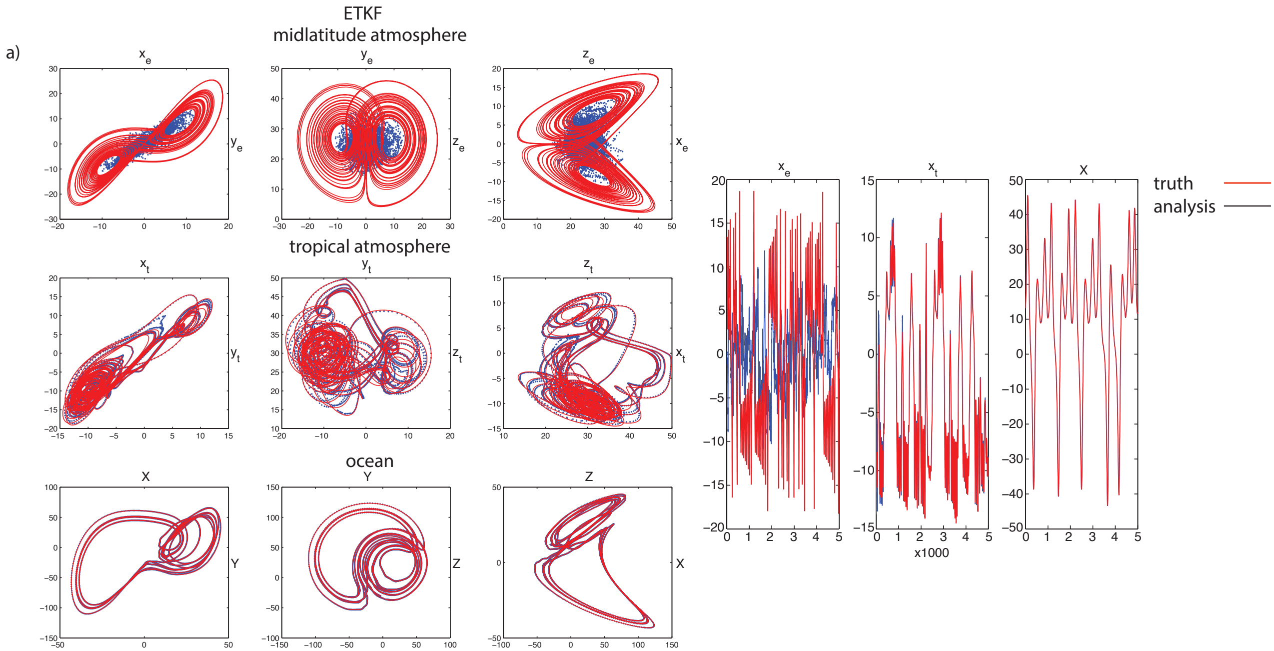
4.2 Paradigm Model

”Ocean” assimilation coupled covariances: Static background covariances
Analysed tropical atmosphere is on a different attractor. Variance of extra-tropical atmosphere unchanged.



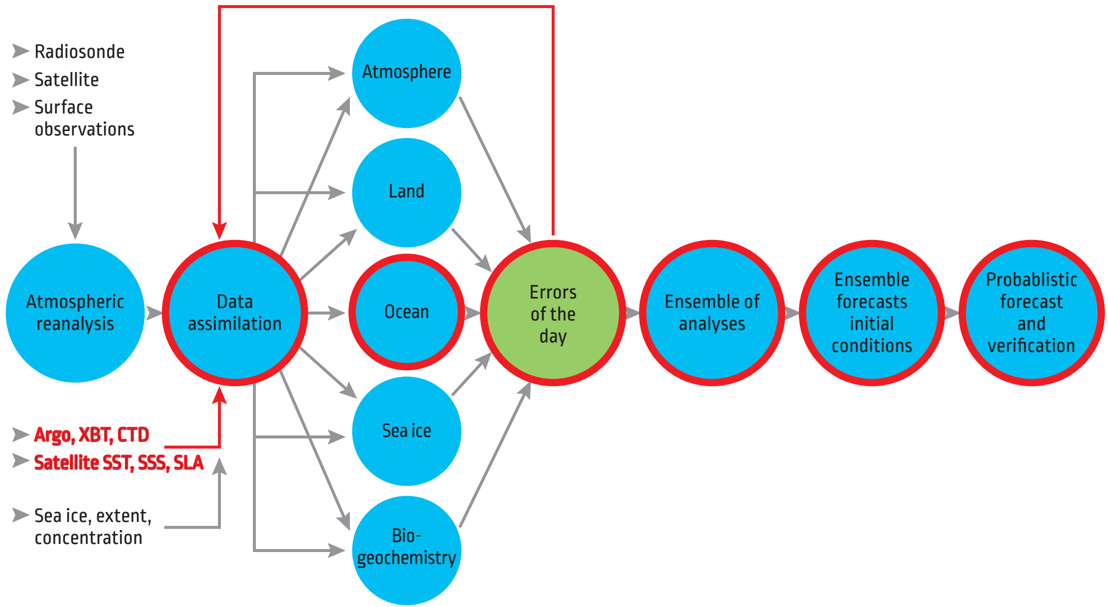
4.3 Paradigm Model

”Ocean” assimilation - coupled covariances: Flow dependent background covariances. Analysed tropical atmosphere is on the attractor. Variance of extra-tropical atmosphere is suppressed.



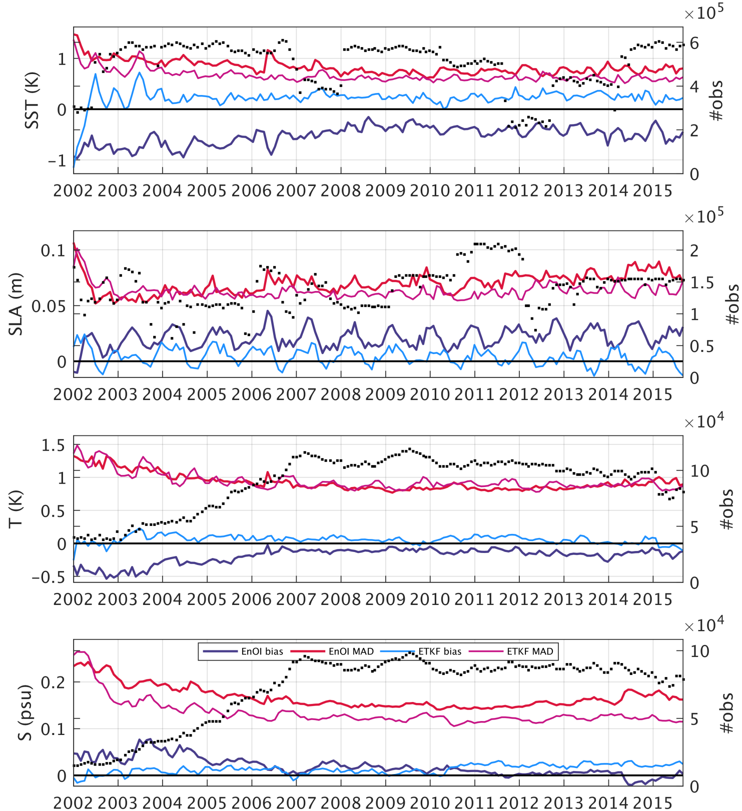
5 CAFE system design

Schematic of the CAFE system



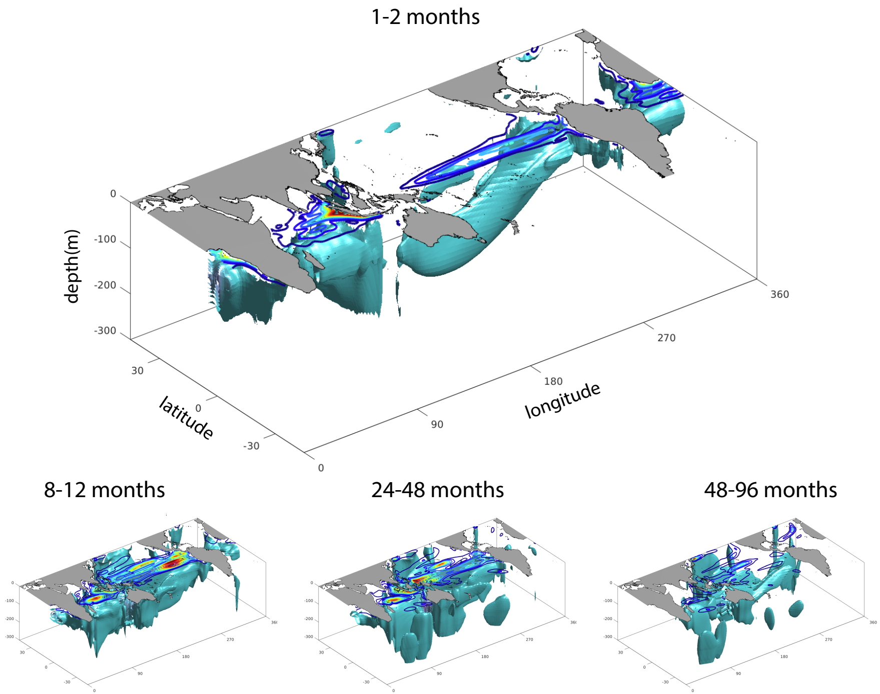
6 Analysed state (20°S-20°N)

"Ocean" assimilation only: Comparison EnOI versus ETKF



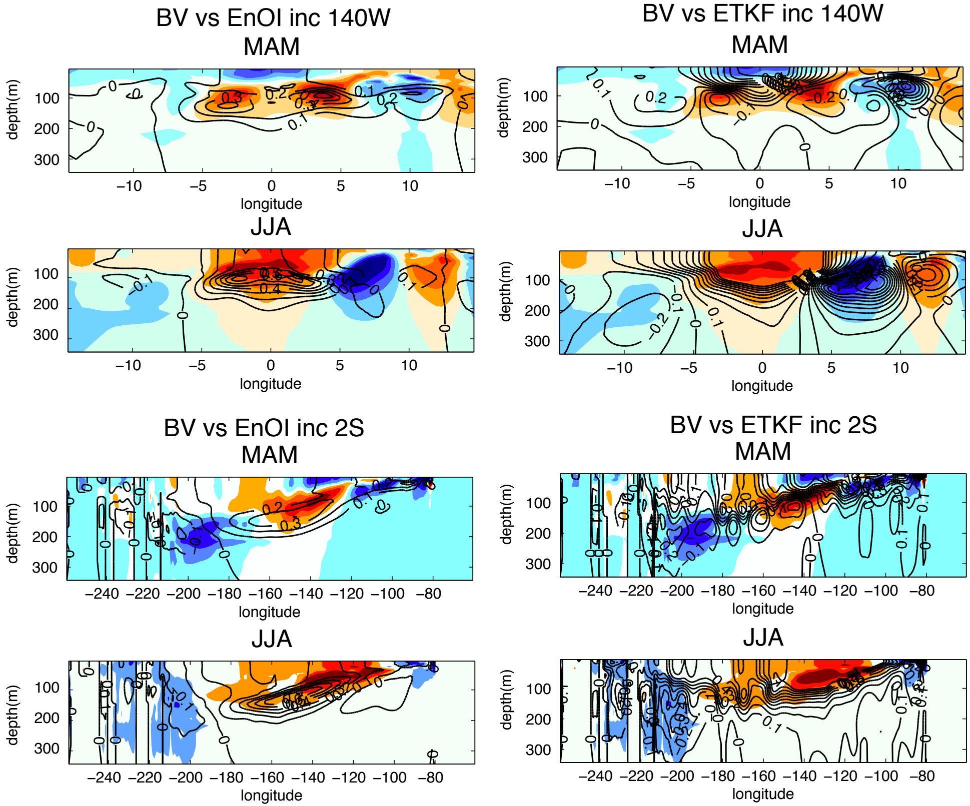
7.1 Scale selection

● localisation length scales and adjustment of observation impact factors to ”tune” increments to select spatio-temporal scales of relevance to given forecast lead times.
● mask regions of variance relevant to those chosen spatio-temporal scales such as the in band variance for temperature with an appropriate threshold (0.5 RMSE calculated from 500 years of control simulation).



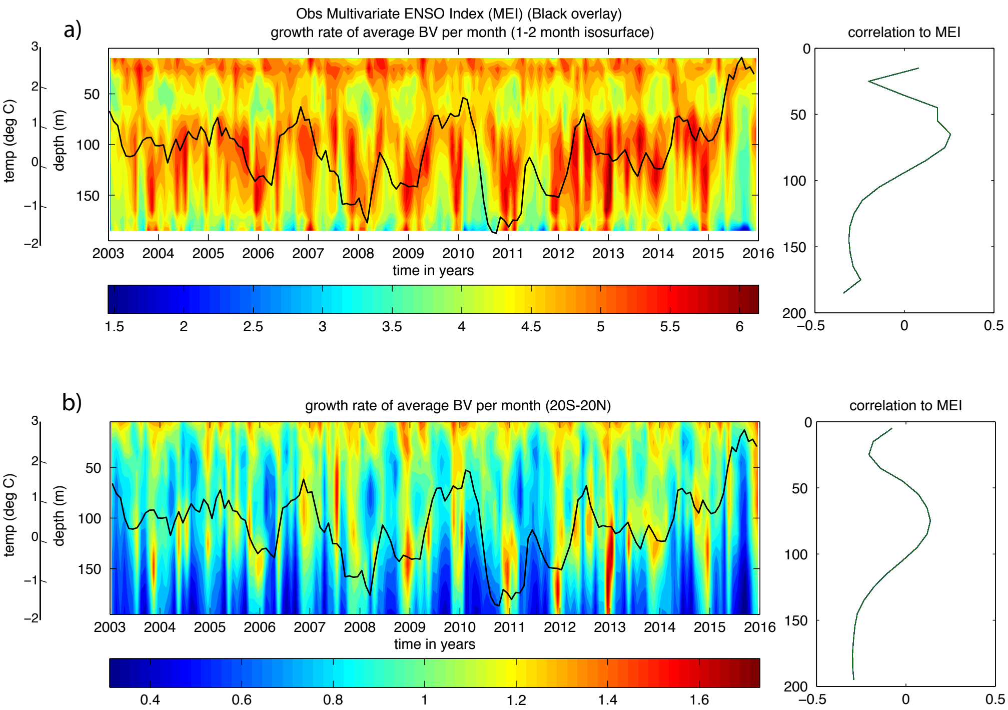
6 Analysed state (20°S-20°N)

Comparison of ensemble averaged BV's (shaded) and EnOI/ETKF analysis increment (contour) along sections at $140°$W and $2°$S.



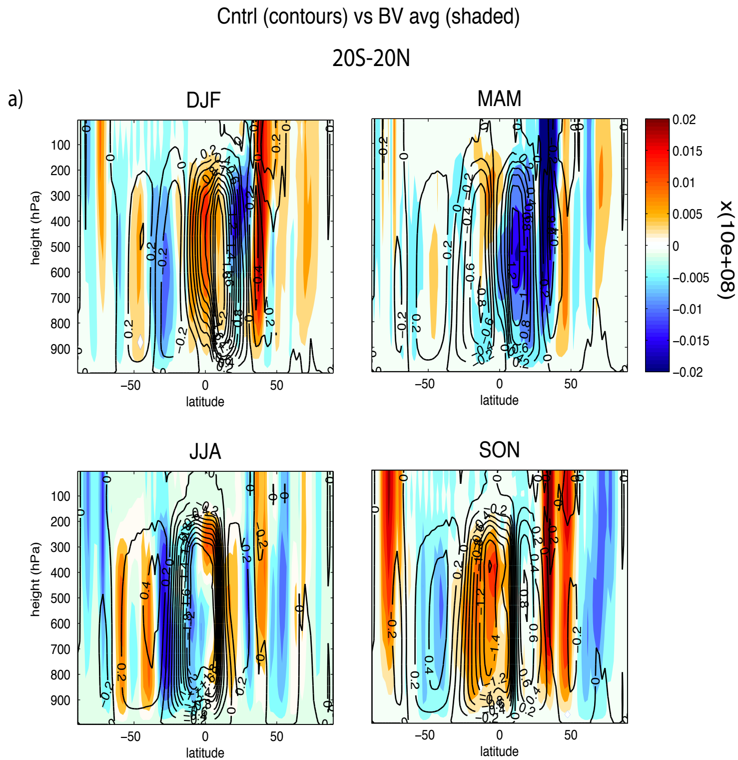
7.2 Scale selection

isosurface BV growth rates 3 times larger than 20°S-20°N BVs



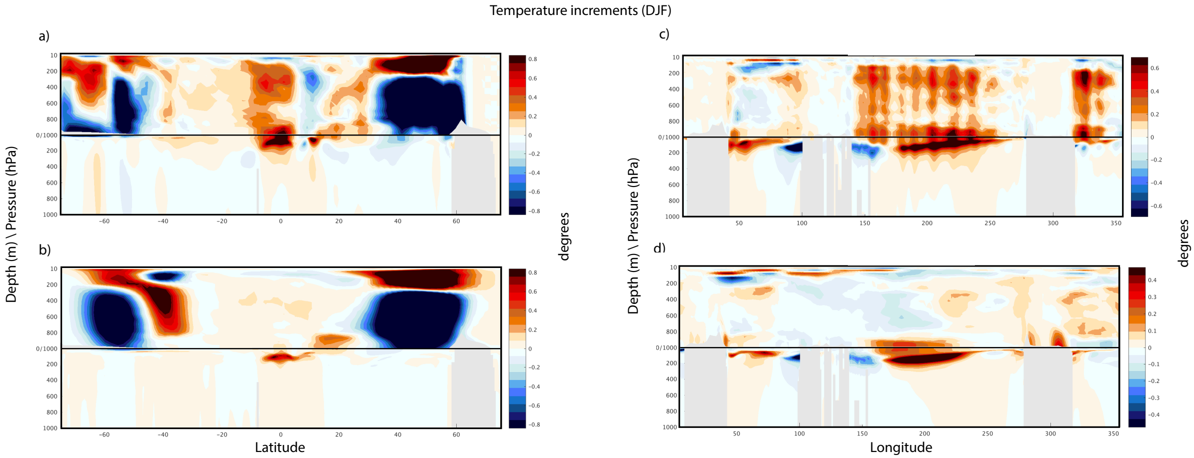
8.2 Tropics-Extra-tropics

Coherent tropospheric response to modulation of tropical convection



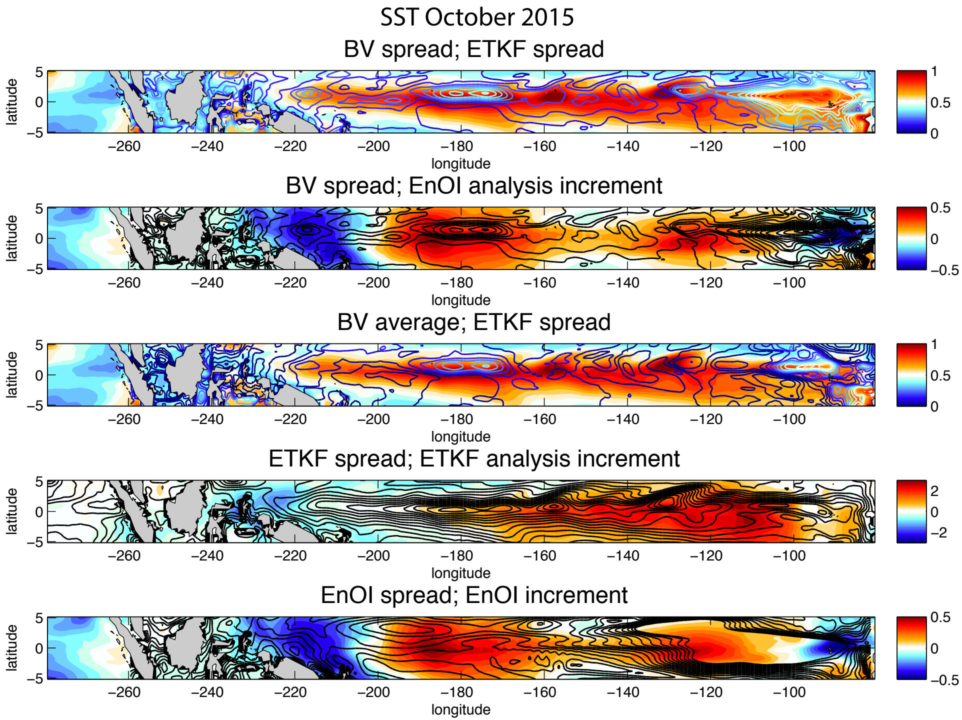
8.1 Tropics-Extra-tropics

Comparison of ocean and atmosphere increments averaged over the boreal winter (DJF) along 140°W (ETKF a); EnOI b) and 2°S (ETKF c); EnOI d)



8.1 ENSO Prediction

- Ensemble spread versus analysis increment during build up to 2016 El Niño.
- BVs add similar flow dependent structures to ETKF background covariances.

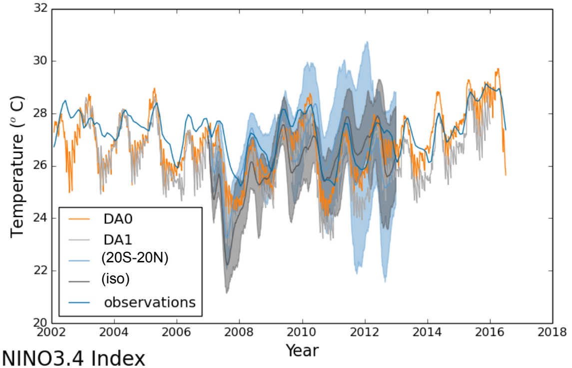


8.2 ENSO Prediction

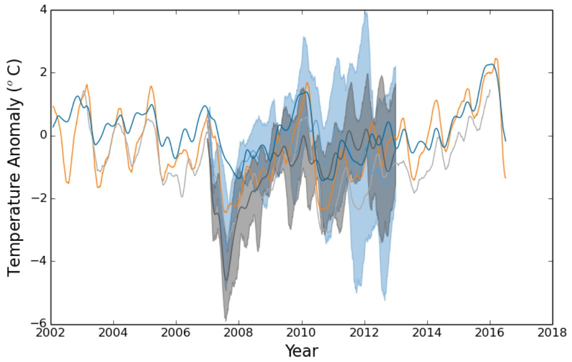
- Ensemble forecasts beginning January 2007 comparing isosurface BVs to BVs generated between 20°N-20°N BVs (renormalised to 1% of the background RMSE).
- Spread reduced in isosurface ensemble due to reduced spurious error growth.
- DA0 & DA1 are reanalysed state estimates.

Note: no SST perturbations are used in isosurface BVs - predictability comes from thermocline perturbations)

NINO3.4 Raw

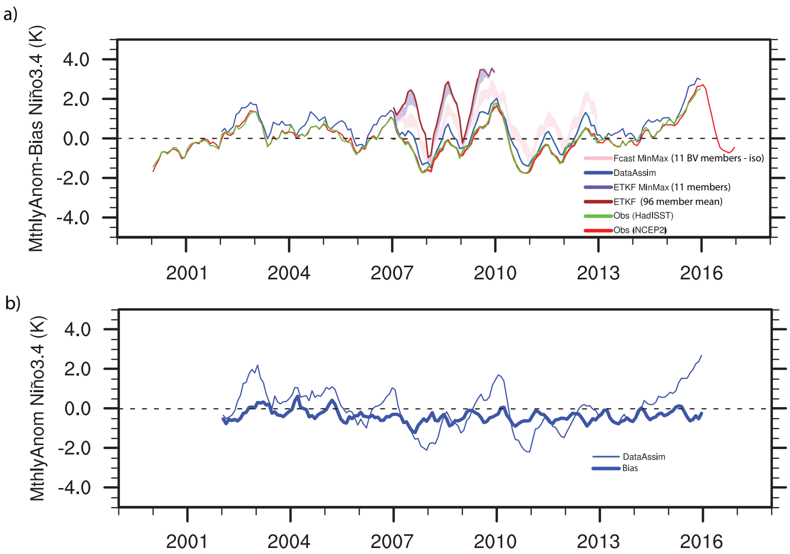


NINO3.4 Index



8.3 ENSO Prediction

a) Bias corrected ETKF and EnOI+isosurface forecasts. b) Analysed state and bias.



Conclusion

- A properly observed ocean is required to constrain the slow climate ”manifold”
- For multi-year forecasting we do not try to track the fast convective or synoptic scales of the atmosphere but rather excite the slow predictable modes coupled to the ocean.
- Optimal perturbations for state estimation are not necessarily optimal for forecasting a given climate mode at a given lead time and should be augmented or replaced by perturbations specific to the phenomena of interest.
- Here we show that it is possible on seasonal timescales to modulate the mid-troposphere jets via targeted perturbations to the tropical thermocline however, how longer timescale memory residing in the subtropical oceans is communicated to, and to he response and predictability of, the atmosphere to these perturbations is still unclear.
- The CAFE system is being developed as a tool to target and understand the mechanisms by which coherent variability determines predictability in the climate system in the near term.

References

- Franzke, C.L.E. & T.J. O’Kane, ”Nonlinear and Stochastic Climate Dynamics”, Cambridge University Press (2017) ISBN 978-1-107-11814-0
- O’Kane, T.J., P.A. Sandery, D.P. Monselesan, P. Sakov, M.A. Chamberlain, R. Matear, & M. Collier (2018) ”Coupled data assimilation and ensemble initialization with application to multi-year ENSO prediction.” (submitted J. Climate)

Thank You

CSIRO Oceans & Atmosphere

Terence O'Kane
t +61 3 6232 5066
e terence.okane@csiro.au
w http://people.csiro.au/O/T/Terence-OKane