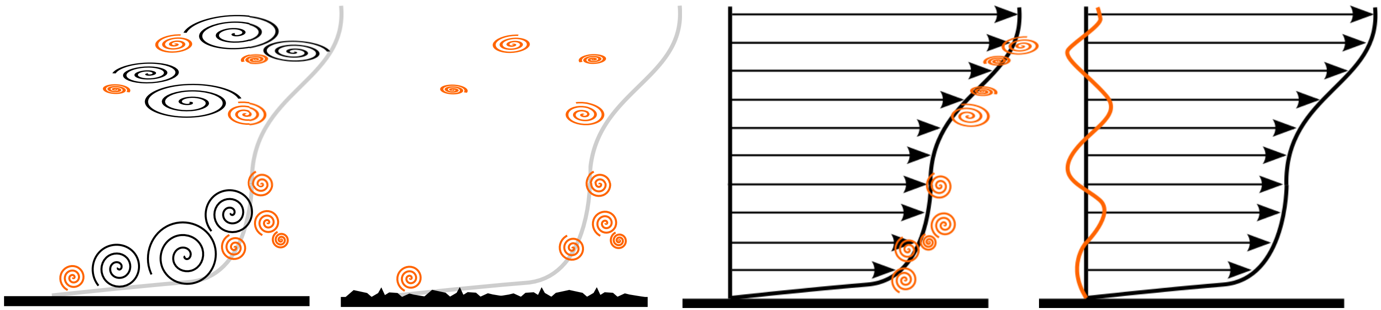

Stochastic subgrid turbulence parameterisation of eddy-eddy, eddy-topographic, eddy-meanfield and meanfield-meanfield interactions

Vassili Kitsios & Jorgen S. Frederiksen

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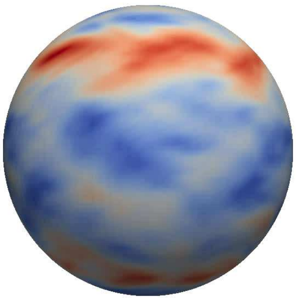
CLIMATE ANALYSIS FORECAST ENSEMBLE system

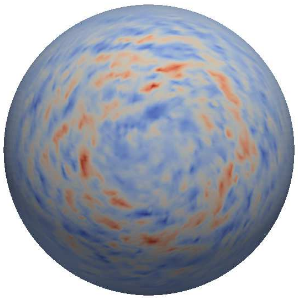
Motivation - Reduce GCM resolution dependence

- Present work focussed on improving accuracy of GCMs, by reducing their resolution dependence.
- It is not possible to simulate all of the scales of motion, hence:
  - the large eddies are resolved by a computational grid
  - unresolved sub-grid scale (SGS) interactions are parameterised
- Typical approach: **Physical Hypothesis** $\rightarrow$ **Subgrid Model**
- Present approach: **Subgrid Model from DNS** $\rightarrow$ **Physical interpretation**
- Present stochastic subgrid modelling approach successfully applied to:



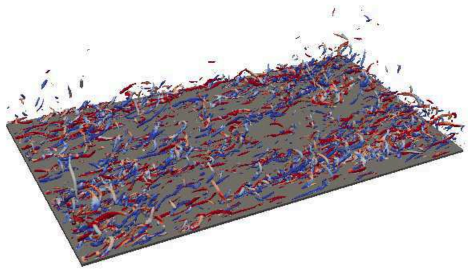
2-level QG Atmos.

Kitsios et. al. (2012, JAS)



2-level QG Ocean

Kitsios et. al. (2013, OM)



300-level Boundary Layer

Kitsios et. al. (2017, C&F)

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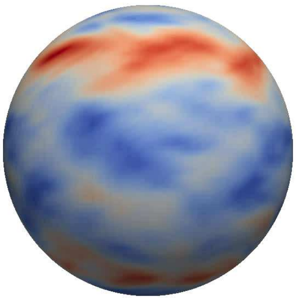

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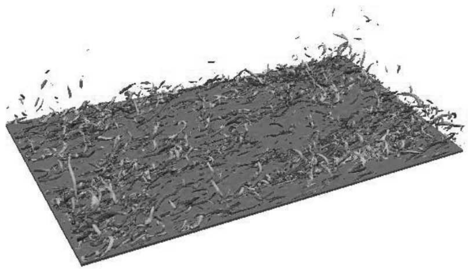


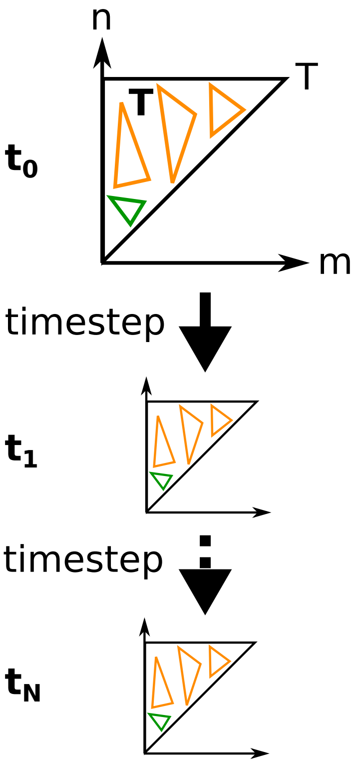
300-level Boundary Layer

Kitsios et. al. (2015, JoT)

Decomposition of Scales: Triangular Truncation

DNS

$$\frac{\partial \mathbf{q}}{\partial t} \equiv \mathbf{q}_t = \mathcal{N}(\mathbf{q}; \mathbf{T})$$



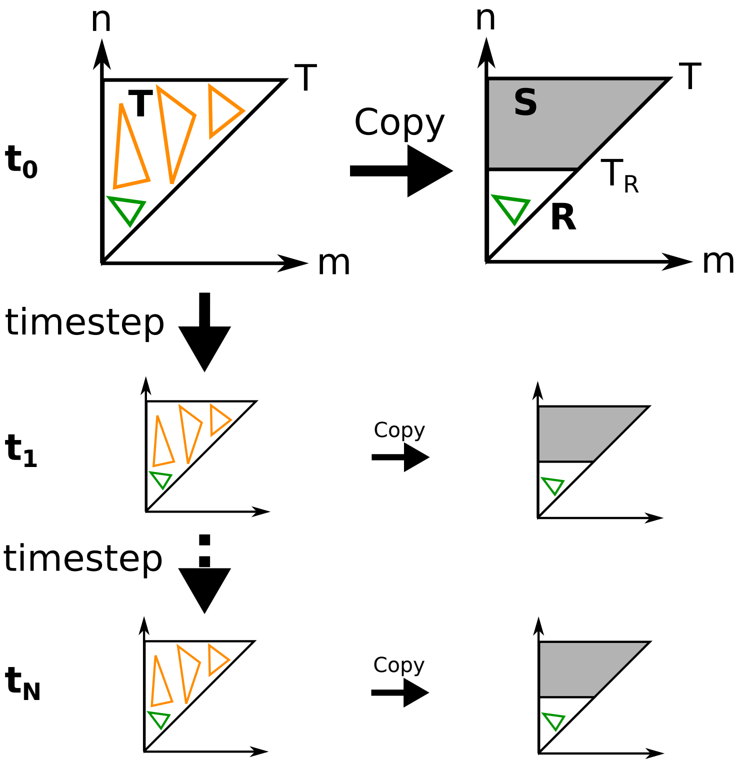
Decomposition of Scales: Triangular Truncation

DNS

$$\frac{\partial \mathbf{q}}{\partial t} \equiv \mathbf{q}_t = \mathcal{N}(\mathbf{q}; \mathbf{T})$$

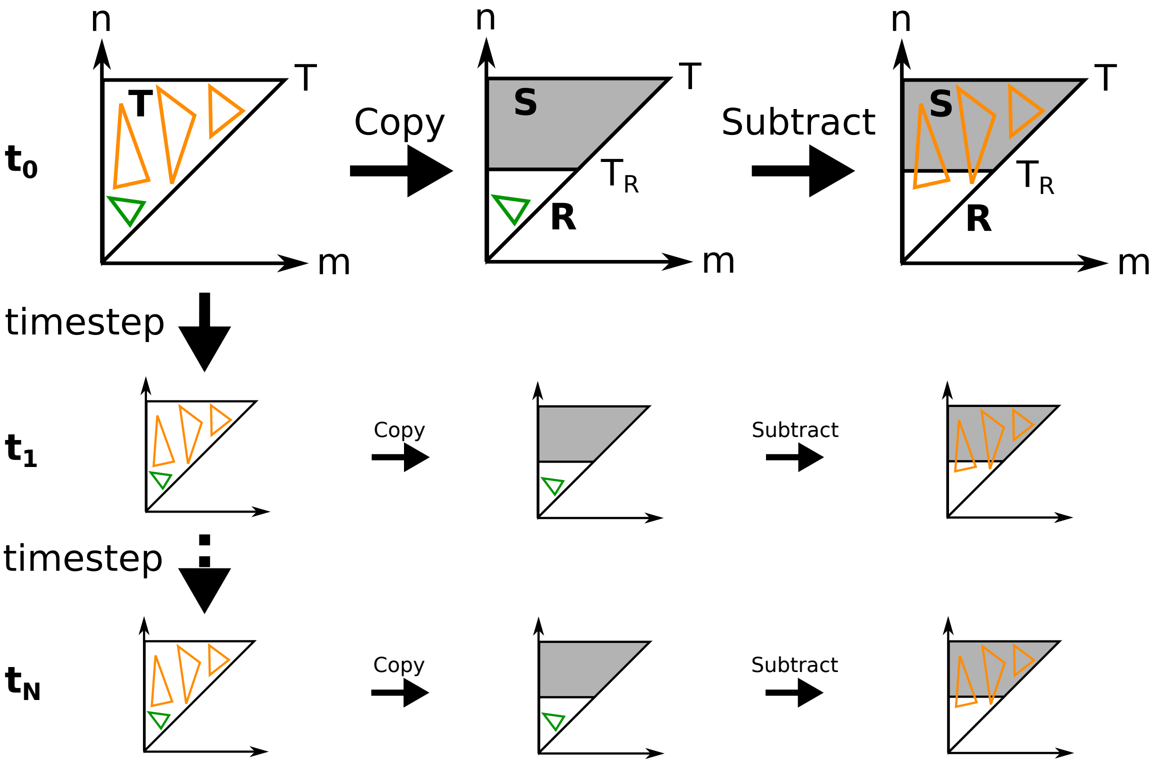
Truncated DNS

$$\mathbf{q}_t^{\mathbf{R}} = \mathcal{N}(\mathbf{q}; \mathbf{R})$$



Decomposition of Scales: Triangular Truncation

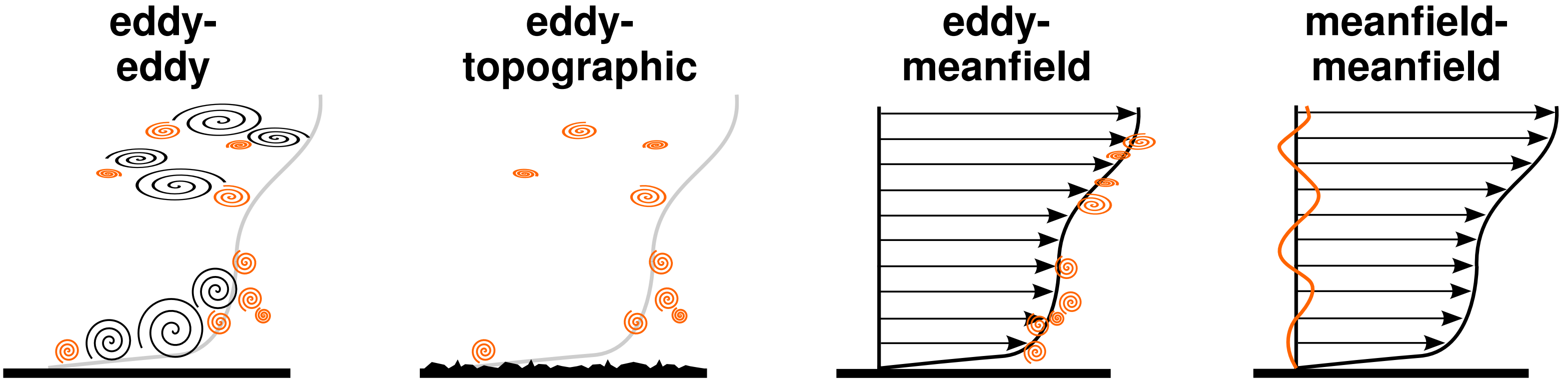
DNS	Truncated DNS	Subgrid Tendency
$\frac{\partial \mathbf{q}}{\partial t} \equiv \mathbf{q}_t = \mathcal{N}(\mathbf{q}; \mathbf{T})$	$\mathbf{q}_t^{\mathbf{R}} = \mathcal{N}(\mathbf{q}; \mathbf{R})$	$\mathbf{q}_t^{\mathbf{S}} = \mathbf{q}_t - \mathbf{q}_t^{\mathbf{R}}$



- In most primitive form, subgrid modelling relates $\mathbf{q}_t^{\mathbf{S}}$ to $\mathbf{q}$.

Subgrid interactions

- **Eddy-eddy**: *subgrid eddies* and *resolved eddies*.
- **Eddy-topographic**: *subgrid eddies* and *resolved topography*.
- **Eddy-meanfield**: *subgrid eddies* and *resolved meanfield*.
- **Meanfield-meanfield**: *subgrid meanfield* and *resolved meanfield*.



$$\mathbf{q}_t^{\mathbf{S}} = \widehat{\mathbf{q}}_t^{\mathbf{S}} + \bar{\mathbf{f}}_{\mathbf{h}} + \bar{\mathbf{f}}_{\mathbf{m}} + \bar{\mathbf{j}}$$

$$-\mathbf{D}_{\text{net}}\hat{\mathbf{q}} \qquad \boldsymbol{\chi} h \qquad -\bar{\mathbf{D}}\bar{\mathbf{q}} \qquad \mathcal{N}(\bar{\mathbf{q}}; \mathbf{S})$$

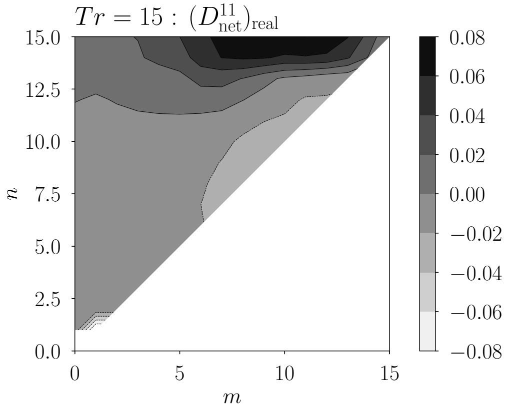
- Functional forms derived from closure theory (Frederiksen, 1999, JAS)

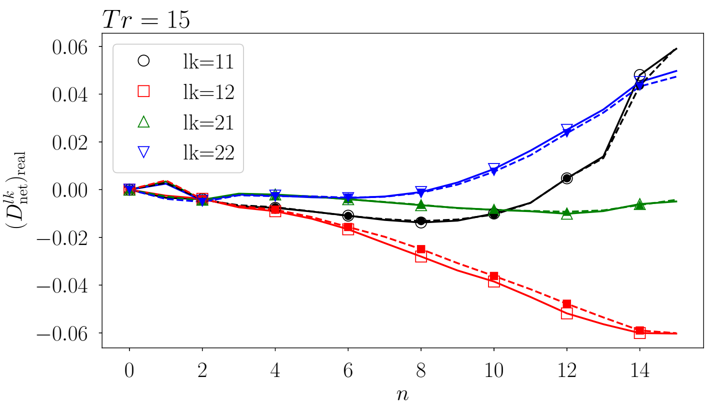
Eddy-Eddy Dissipation

- Subgrid interactions are *local* in wavenumbers space, not grid space.
- Unique Matrix captures level interactions per scale (Frederiksen & Kepert, 2006)

Deterministic	**Stochastic**
$\widehat{\mathbf{q}}_t^{\mathbf{S}}(t) = -\mathbf{D}_{\mathbf{net}}\widehat{\mathbf{q}}(t)$	$\widehat{\mathbf{q}}_t^{\mathbf{S}}(t) = -\mathbf{D}_{\mathbf{d}}\widehat{\mathbf{q}}(t) + \widehat{\mathbf{f}}(t)$
$\left\langle \widehat{\mathbf{q}}_t^{\mathbf{S}}(t)\widehat{\mathbf{q}}^{\dagger}(t)\right\rangle = -\mathbf{D}_{\mathbf{net}}\left\langle \widehat{\mathbf{q}}(t)\widehat{\mathbf{q}}^{\dagger}(t)\right\rangle$	$\mathbf{D}_{\mathbf{d}}$ incorporates history
$\mathbf{D}_{\mathbf{net}} = -\left\langle \widehat{\mathbf{q}}_t^{\mathbf{S}}(t)\widehat{\mathbf{q}}^{\dagger}(t)\right\rangle \left\langle \widehat{\mathbf{q}}(t)\widehat{\mathbf{q}}^{\dagger}(t)\right\rangle^{-1}$	$\mathbf{D}_{\mathbf{b}} \propto$ variance of $\widehat{\mathbf{f}}(t)$

- At $T_R = 15$ baroclinic instability is not resolved
- Dissipation anisotropic, negative for many scales, off diagonals significant





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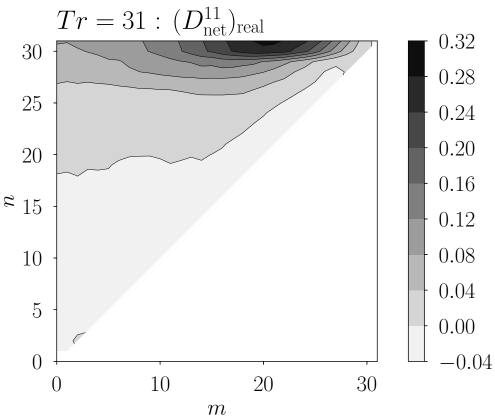
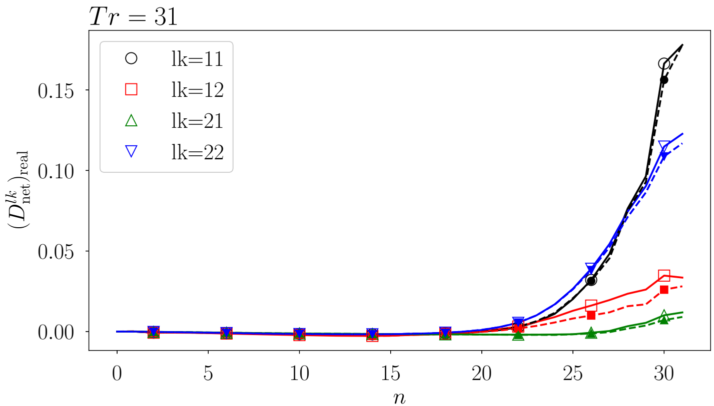
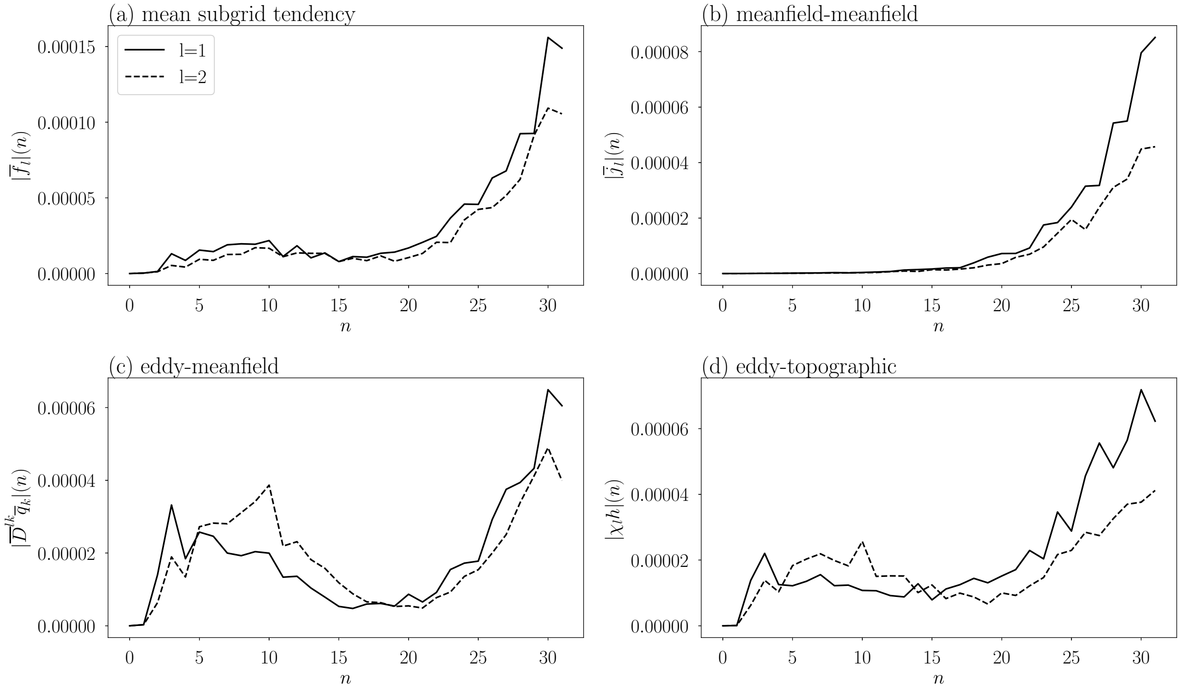
- At $T_R = 31$ baroclinic instability is better resolved
- Dissipation isotropic, negative for fewer scales, off diagonals smaller





Decompose Mean Subgrid Tendency $\equiv \overline{\mathbf{q}_t^{\mathrm{S}}} \equiv \bar{\mathbf{f}}_{\mathrm{h}} + \bar{\mathbf{f}}_{\mathrm{m}} + \bar{\mathbf{j}}$

- **meanfield-meanfield**: evaluate $\bar{\mathbf{j}} = \mathcal{N}(\bar{\mathbf{q}}; \mathbf{S}) \equiv \mathcal{N}(\bar{\mathbf{q}}; \mathbf{T}) - \mathcal{N}(\bar{\mathbf{q}}; \mathbf{R})$
- **eddy-meanfield**: $\bar{\mathbf{f}}_{\mathrm{m}} = -\bar{\mathrm{D}}\bar{\mathbf{q}}$
  determine $\bar{\mathrm{D}}$ via regression of $[\overline{\mathbf{q}_t^{\mathrm{S}}} - \bar{\mathbf{j}}]$ with $\bar{\mathbf{q}}$ over 500 climate states
- **eddy-topographic**: determined by the remainder $\bar{\mathbf{f}}_{\mathrm{h}} = \overline{\mathbf{q}_t^{\mathrm{S}}} - \bar{\mathbf{j}} + \bar{\mathrm{D}}\bar{\mathbf{q}}$



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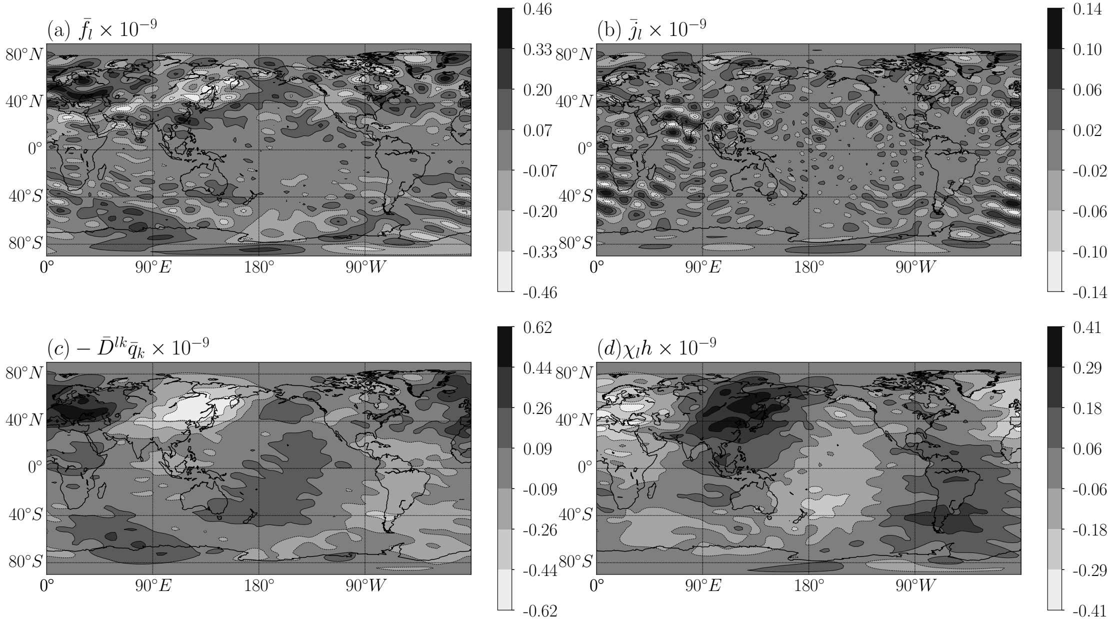
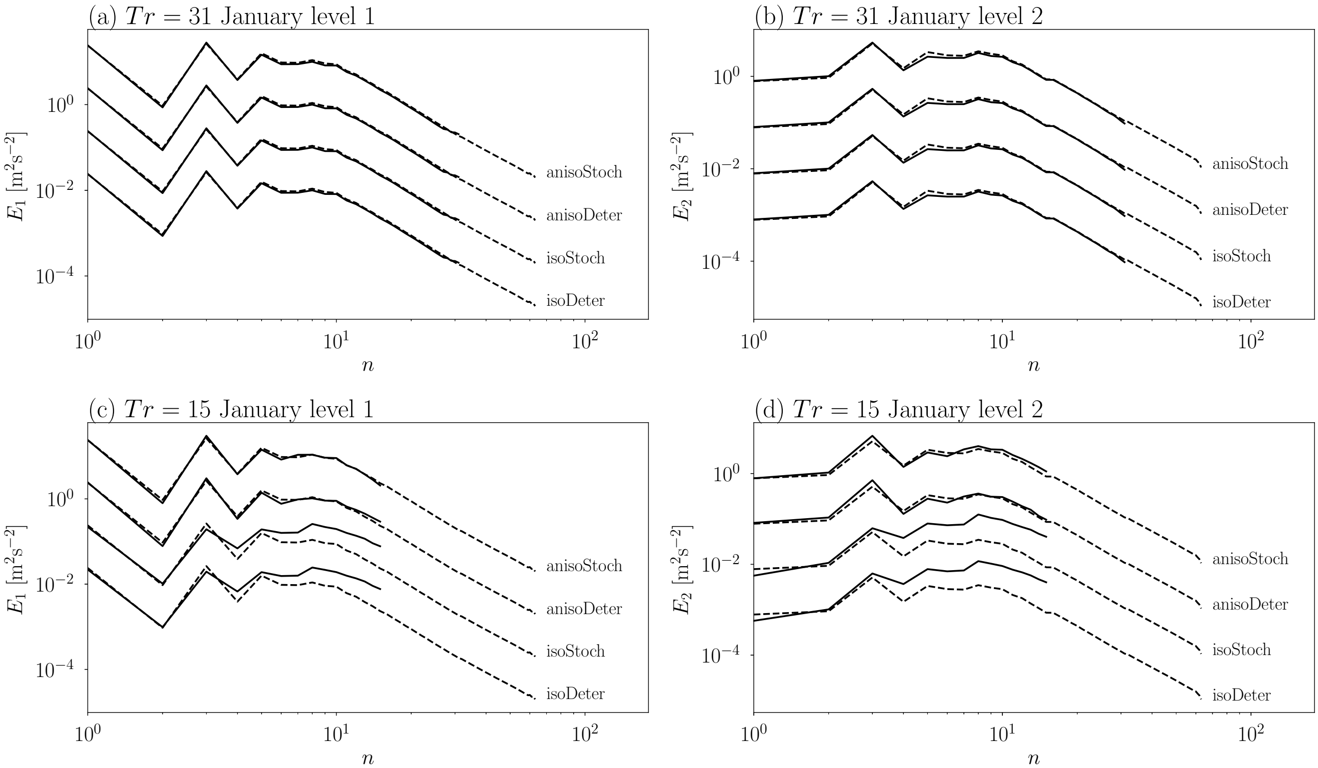


- Similar to previous evaluation of closure terms (O'Kane & Frederiksen, 2008)

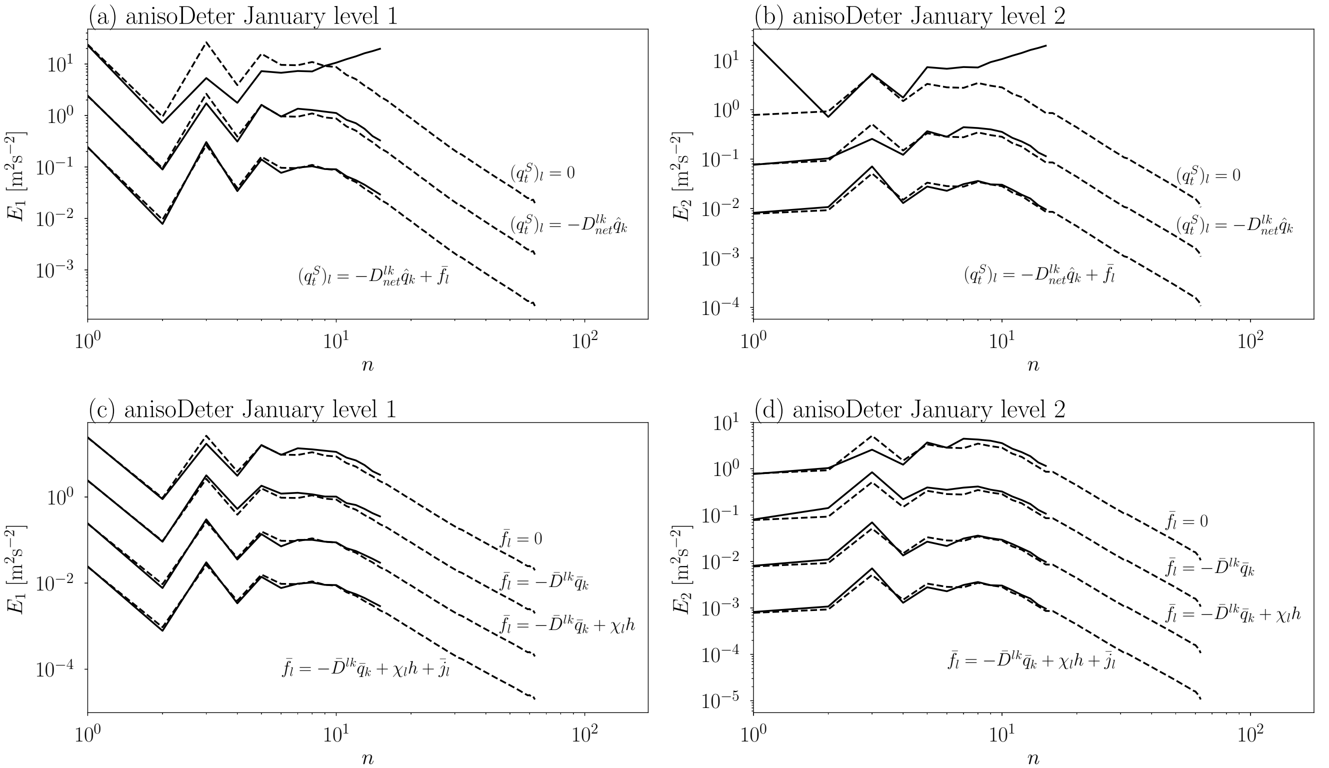
LES : Eddy-Eddy Variants ; Full Mean Subgrid Tendency

- For $T_R = 31$ **stochastic** and **deterministic** variants with **anisotropic** and **isotropic** eddy-eddy coefficients reproduce the DNS for all scales
- For $T_R = 15$ **anisotropic** coefficients are required



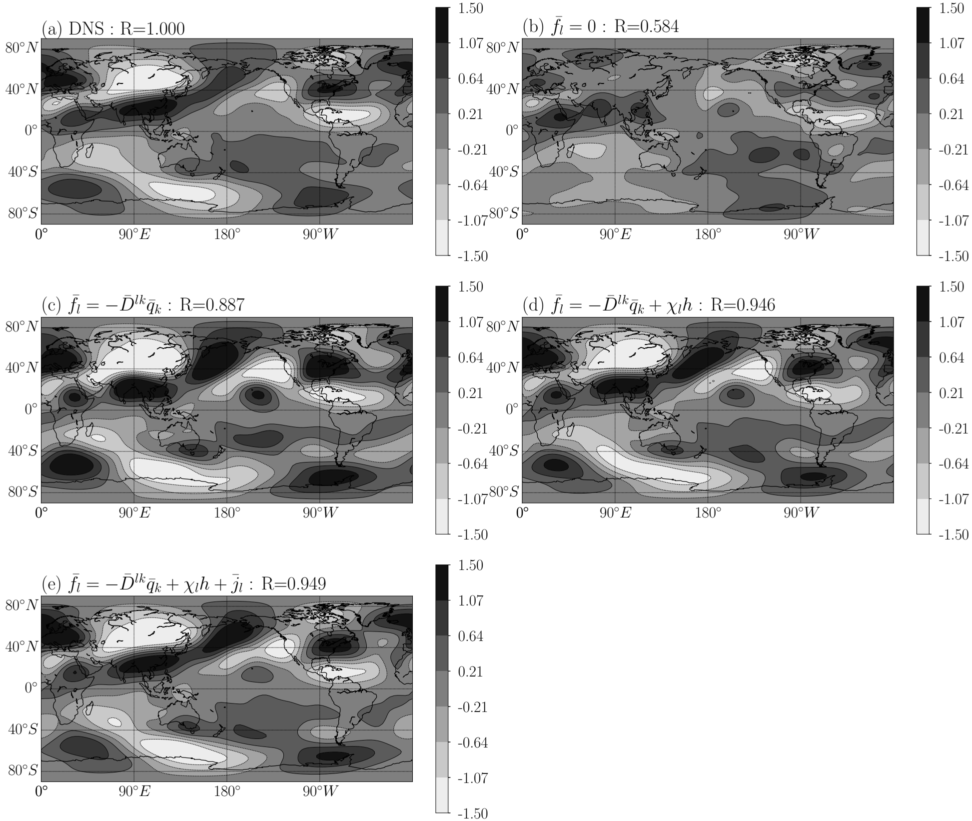
Anisotropic Deterministic LES at $T_R = 15$

- Eddy-eddy parameterisation has dominant impact
- Best agreement requires parameterisation of all subgrid interactions



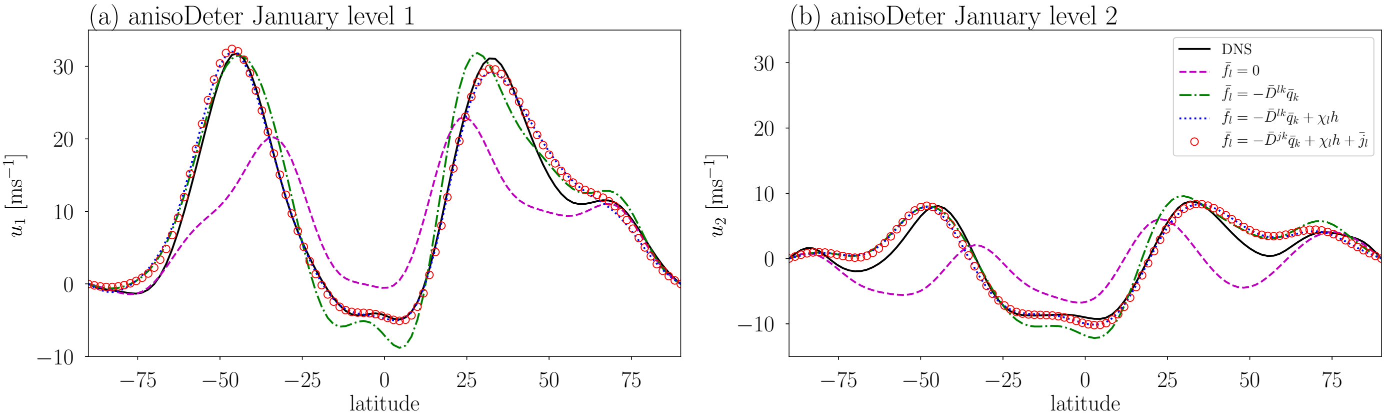
Anisotropic Deterministic LES at $T_R = 15$

- Likewise best pattern correlation of mean nonzonal streamfunction requires parameterisation of all subgrid interactions



Anisotropic Deterministic LES at $T_R = 15$

- Likewise best pattern correlation of mean nonzonal streamfunction requires parameterisation of all subgrid interactions
- Also true to for mean zonal velocity



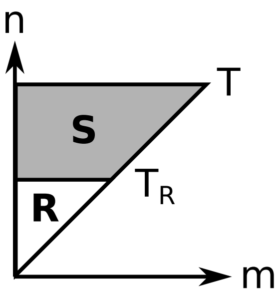
Concluding Remarks

- Stochastic subgrid modelling approach used to determine parameterisation coefficients representing the interactions between the:
  - subgrid eddies and resolved eddies (**eddy-eddy**)
  - subgrid eddies and resolved topography (**eddy-topographic**)
  - subgrid eddies and resolved meanfield (**eddy-meanfield**)
  - subgrid meanfield and resolved meanfield (**meanfield-meanfield**)
- Note there are no tuning parameters in this approach.

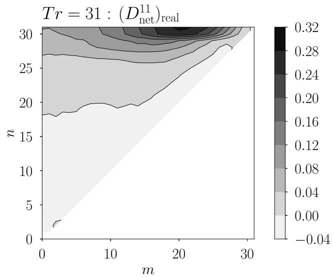
- The eddy-eddy parameterisation has the dominant contribution to reproducing the DNS spectra across all scales.
- However, parameterisation of **all subgrid interactions** required for best agreement of the DNS and LES in terms of:
  - kinetic energy spectra
  - mean nonzonal streamfunction
  - mean zonal velocity

Questions

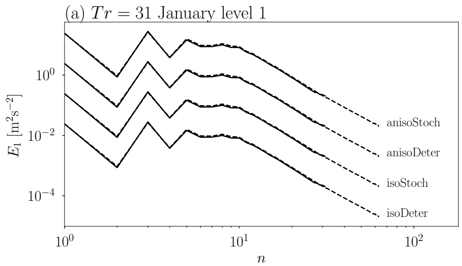
1) Decompose DNS



2) Calculate Dissipation



3) Verify with LES



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Thank You

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