



Numerical tracer mixing in the COSIMA model suite

Never Stand Still

Science

Climate Change Research Centre



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Introduction

Numerical mixing – artificial diffusion of tracer gradients arising from discrete advection

Contributes to model biases – **“the models mix too much”**.
Interior diapycnal diffusivity is small ($<10^{-5}\text{m}^2\text{s}^{-1}$), spurious numerical mixing can often be larger.

Difficult to quantify in realistic models – typically studied in idealized contexts

This study:

- Numerical mixing estimated across ACCESS-OM2 suite
- Sensitivity to resolution and physical parameterizations.

We do not:

- Test sensitivity to advection schemes (take MDPPM as given)
- Precisely decompose isopycnal vs. diapycnal components

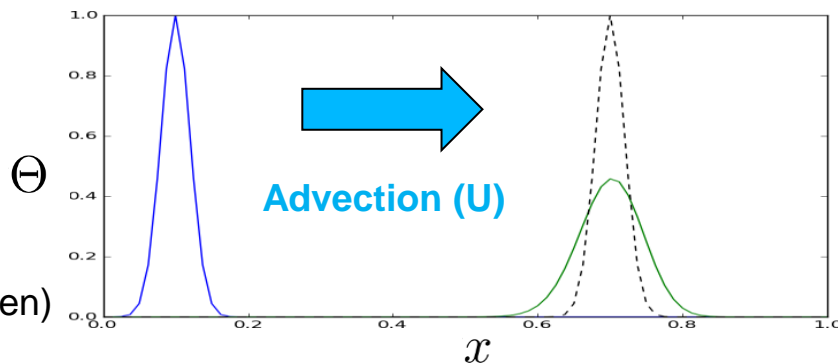
Continuous 1D
Advection:

$$\frac{\partial \Theta}{\partial t} = -U \frac{\partial \Theta}{\partial x}$$

Discrete analog (centered-space, forward-time):

$$\frac{\partial \Theta}{\partial t} = -U \frac{\partial \Theta}{\partial x} + \boxed{U^2 \frac{\Delta t}{2} \frac{\partial^2 \Theta}{\partial x^2}} + \mathcal{O}((\Delta x)^2, (\Delta t)^2)$$

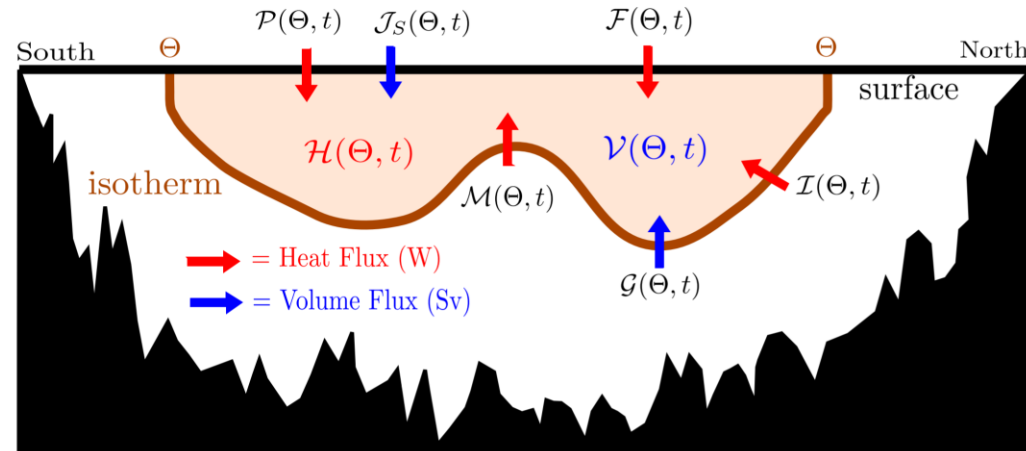
Numerical Diffusion



Method: Heat budget analysis in temperature coordinates

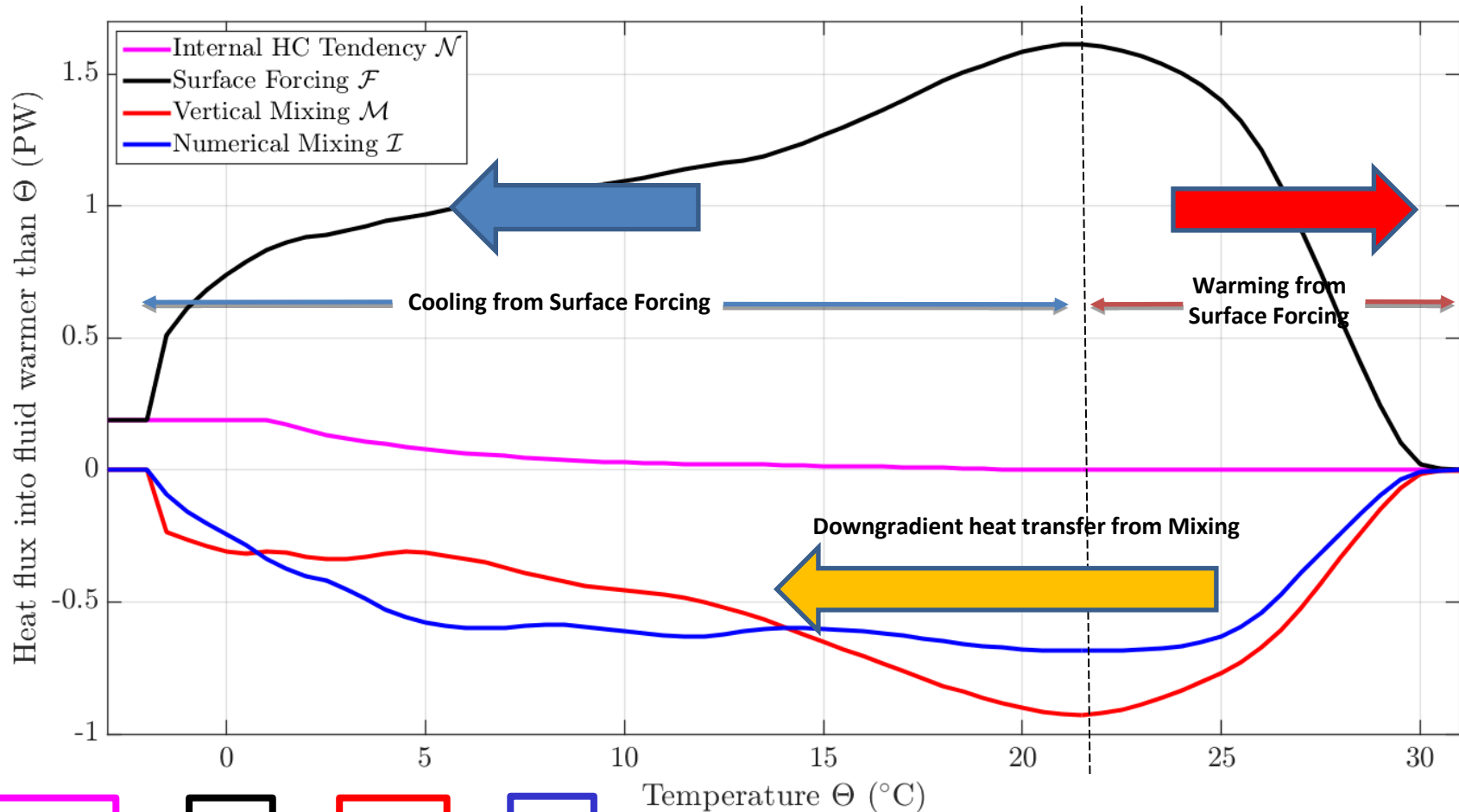
Heat budget of the seawater warmer than Theta:

$$\underbrace{\frac{\partial \mathcal{H}}{\partial t}(\Theta, t)}_{\text{Tendency}} = \underbrace{\mathcal{F}}_{\text{Forcing}} + \underbrace{\mathcal{M}}_{\text{Vertical}} + \underbrace{\mathcal{R}}_{\text{Redi}} + \underbrace{\mathcal{I}}_{\text{Numerical}} + \underbrace{\mathcal{G}\Theta\rho_0 C_p}_{\text{Diathermal Advection}}$$



- Diathermal advection removed by combining with the volume budget – *integrated, more robust*.
- F, M and R calculated by **online** binning Eulerian budget into temperature coordinates. dH/dt tracked using snapshots.
- Numerical mixing (I) calculated by residual

Global Diathermal Heat Budget in MOM025 Control



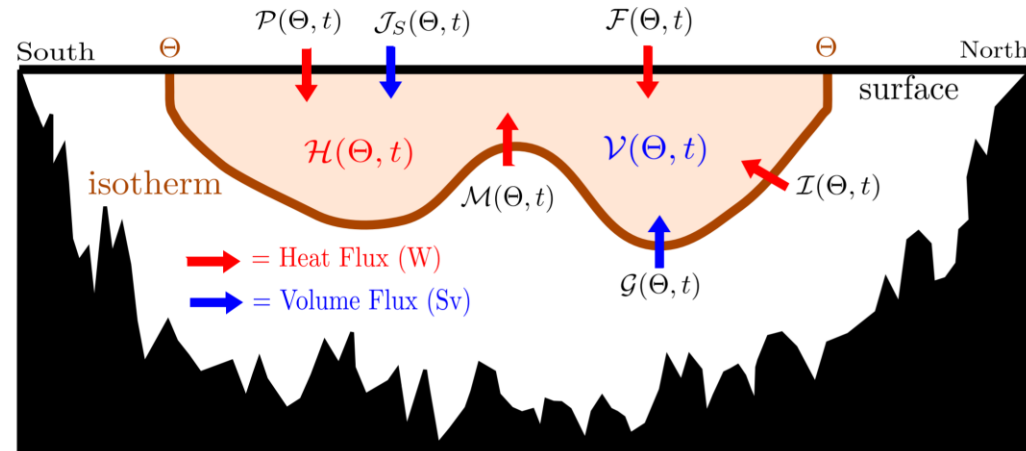
$$\frac{\partial \mathcal{H}_I}{\partial t} = \mathcal{F} + \mathcal{M} + \mathcal{I}$$

**Warm-to-cold heat transport
driven as much by
numerical as vertical mixing**

Method: Heat budget analysis in temperature coordinates

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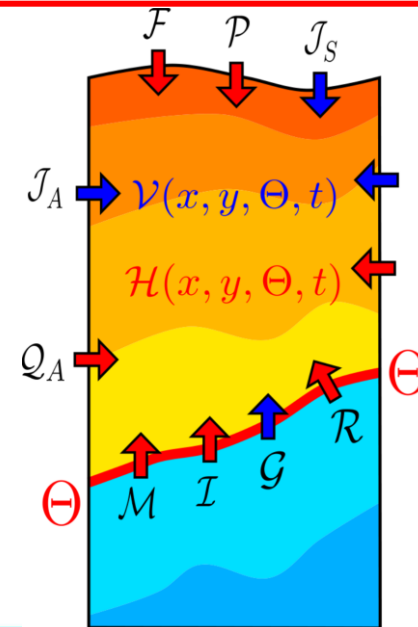


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Spatial Structure

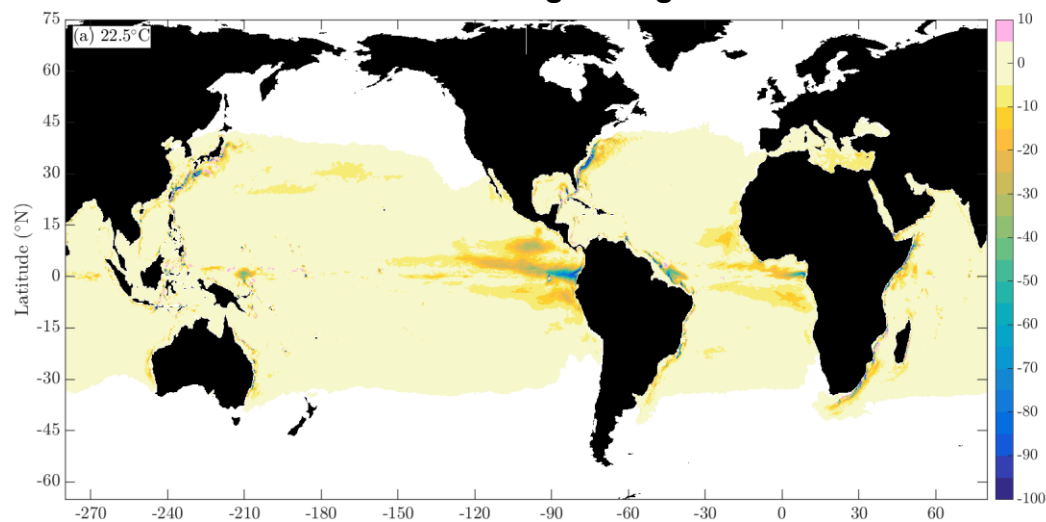
Can be estimated by generalizing to a single fluid column by including temperature-binned lateral volume (J) and heat (Q) fluxes

However – more approximations needed.

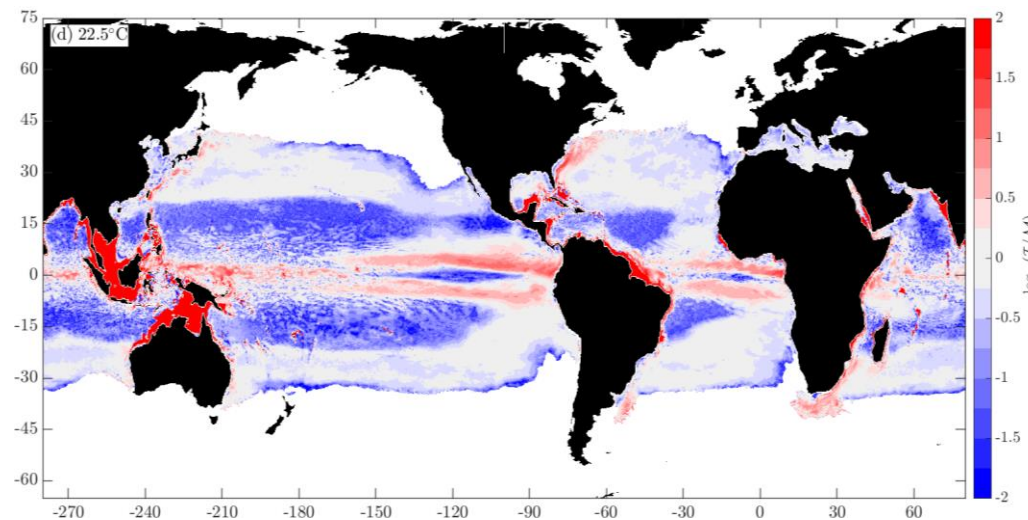


MOM025 Control results -> Warm temperatures

Heat flux due to numerical mixing through 22.5°C isotherm:



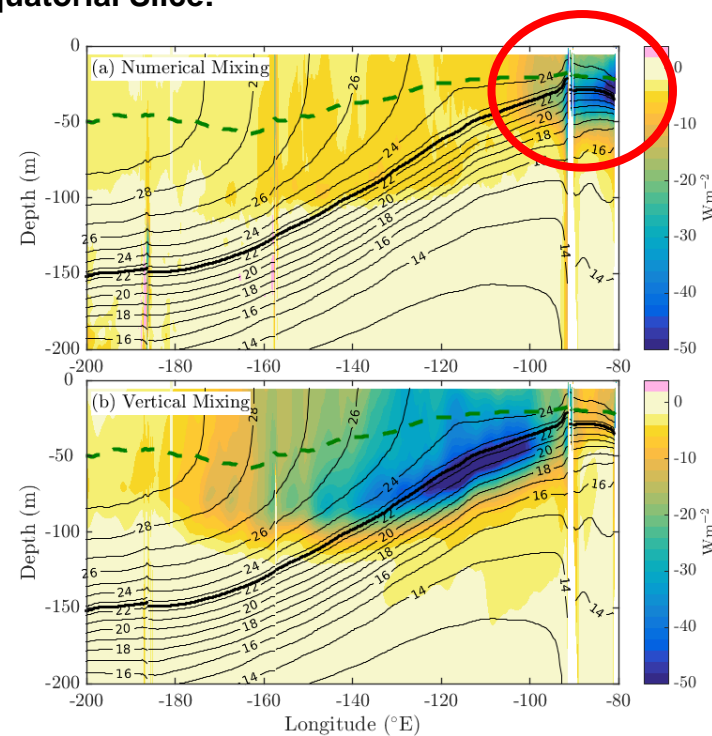
Ratio of numerical mixing to vertical mixing:



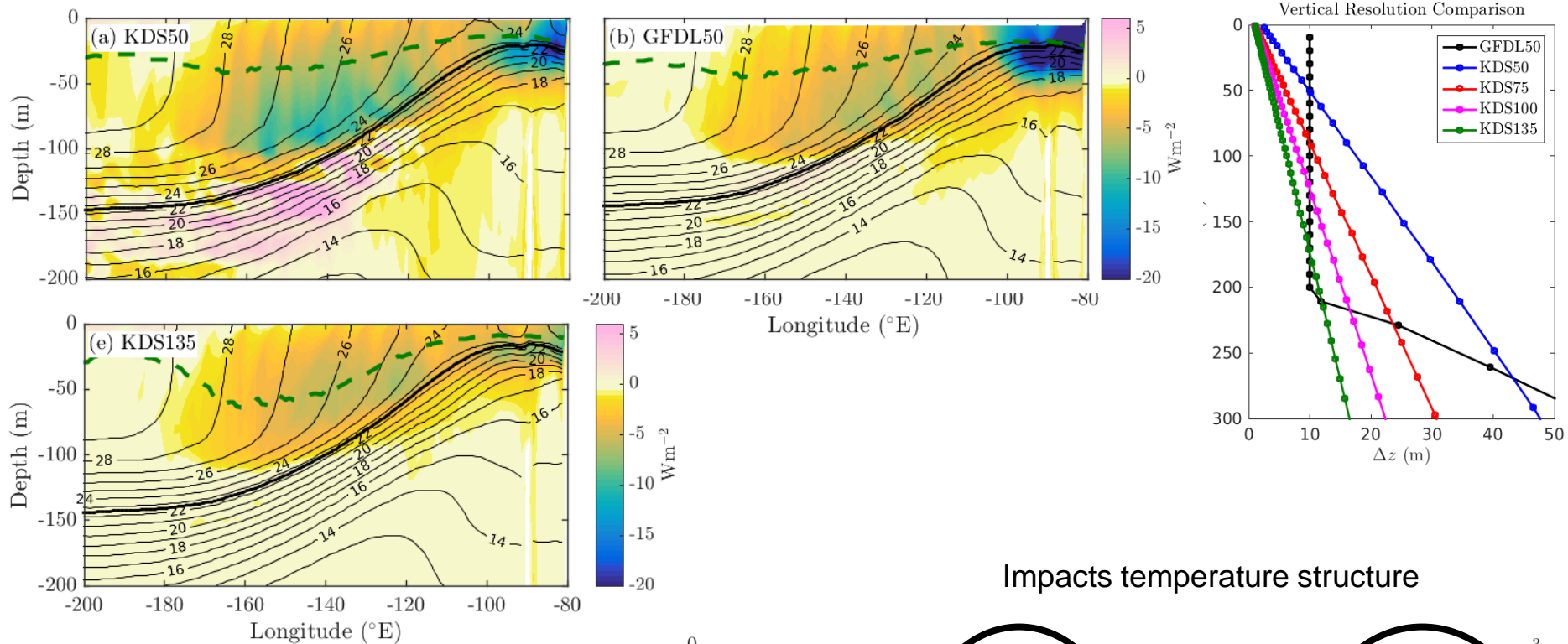
Note: Background diffusivity = $10^{-5} m^2 s^{-1}$
($10^{-6} m^2 s^{-1}$ near Equator)

High grid-scale vertical temperature gradients

Equatorial Slice:



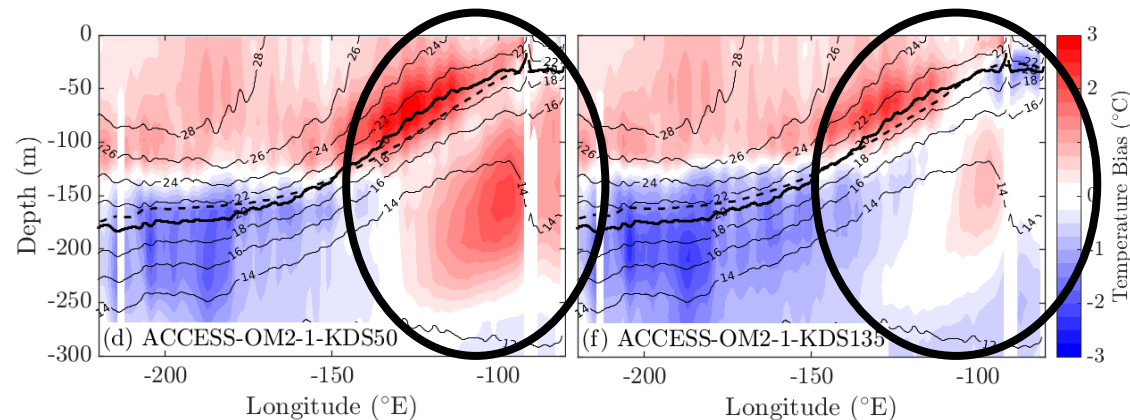
Sensitivity to vertical resolution in ACCESS-OM2-1



Impacts temperature structure

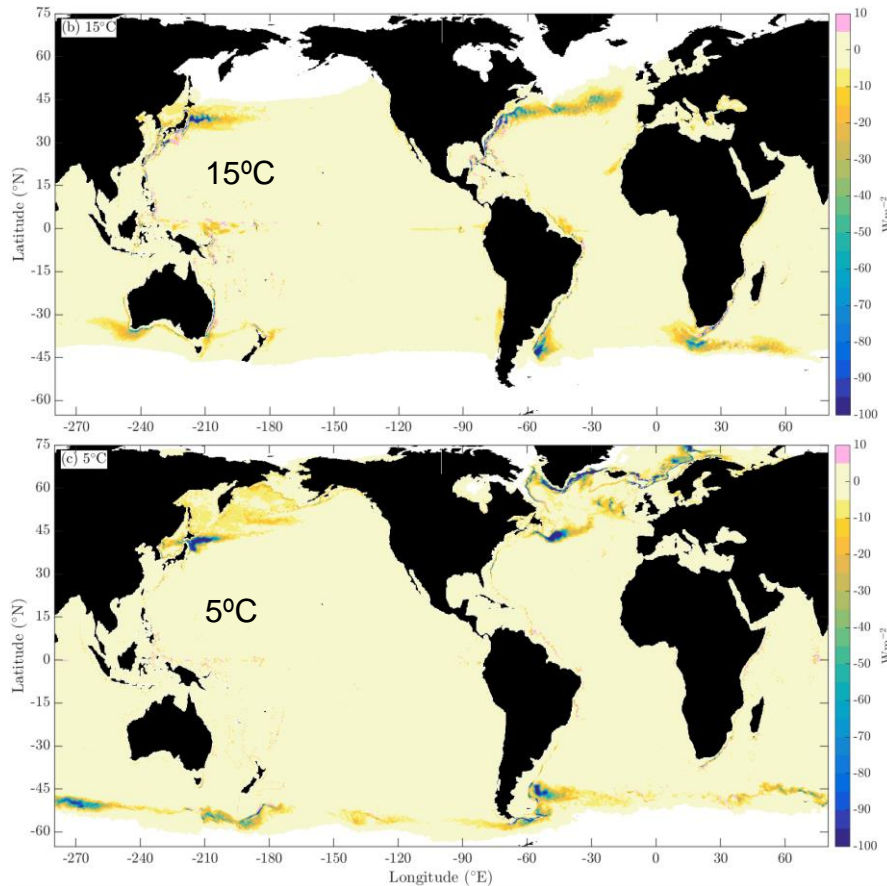
Changes not compensated by changes in vertical mixing.

Sensitivity of numerical mixing at warm temperatures to vertical resolution and vertical diffusivity suggests it is **spurious/diapycnal**.

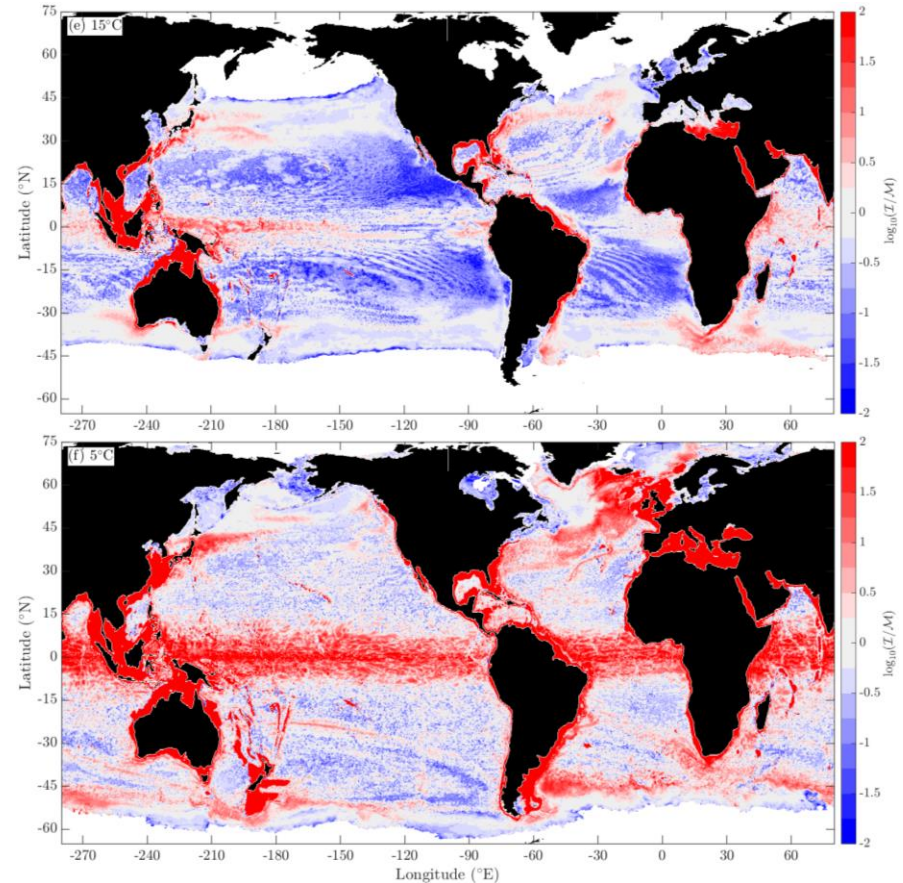


MOM025 Control results -> Cold temperatures

Numerical mixing



Ratio of numerical and vertical



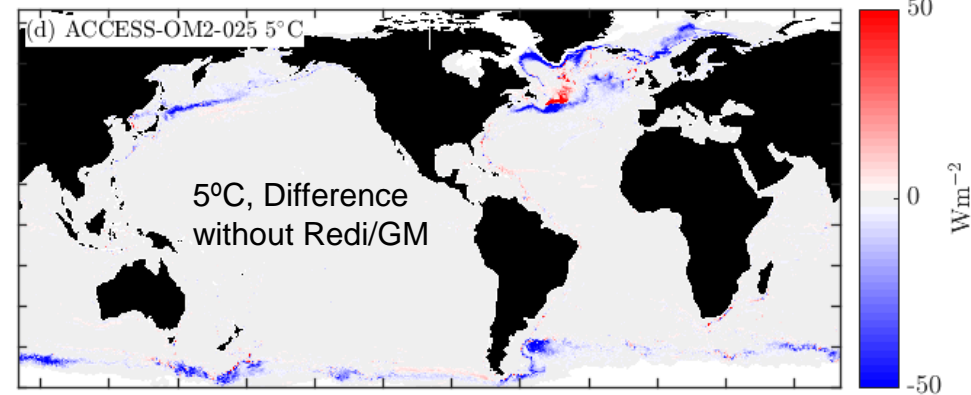
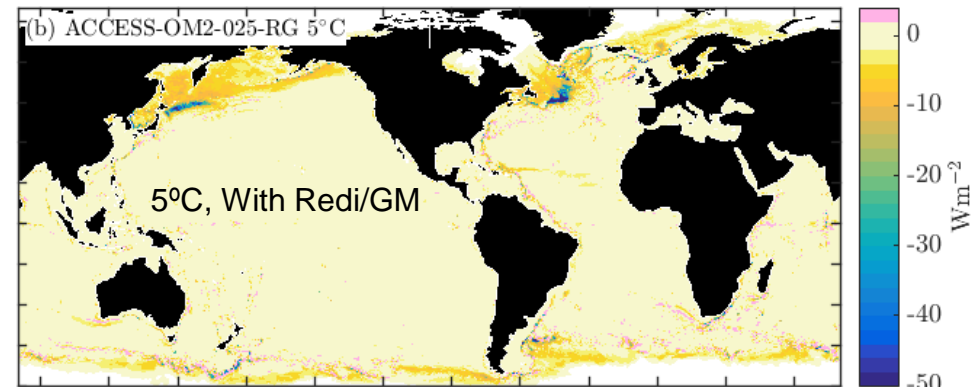
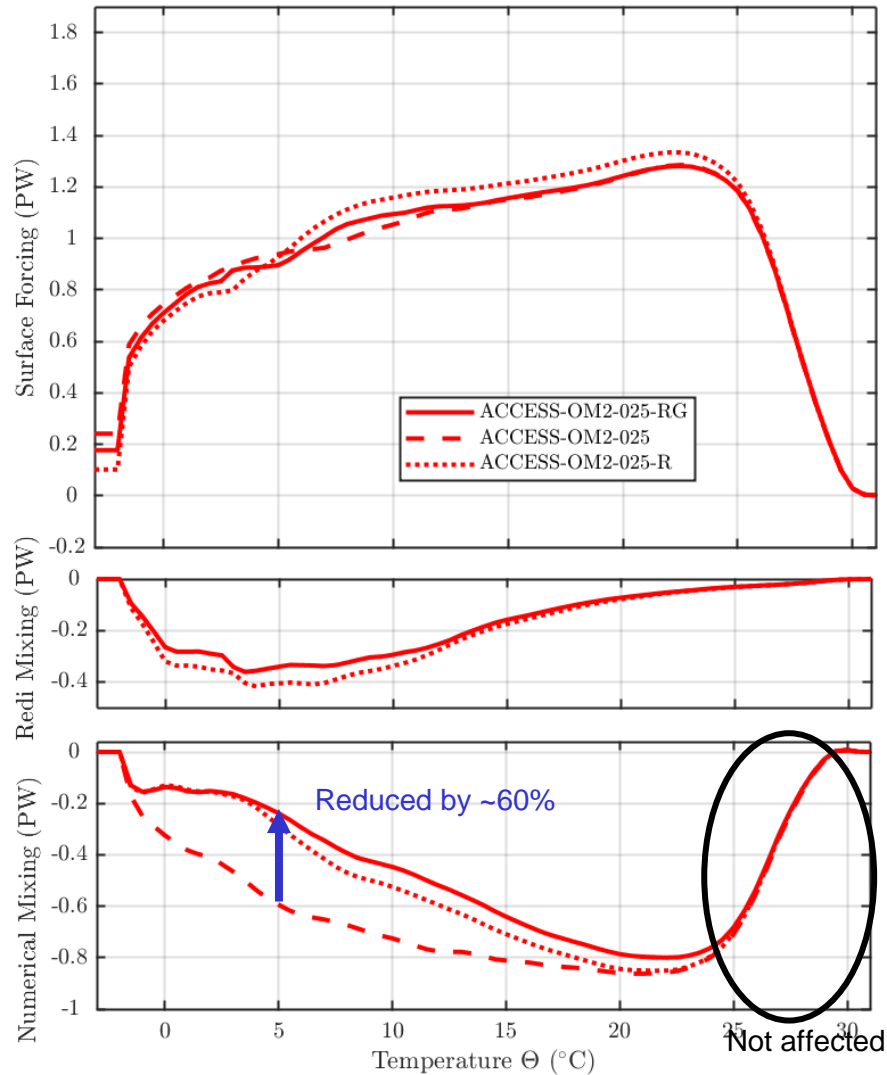
Dominated by eddy-regions – boundary currents and ACC.

Does this mean that numerical mixing is mostly isopycnal in these regions?

Sensitivity to neutral physics

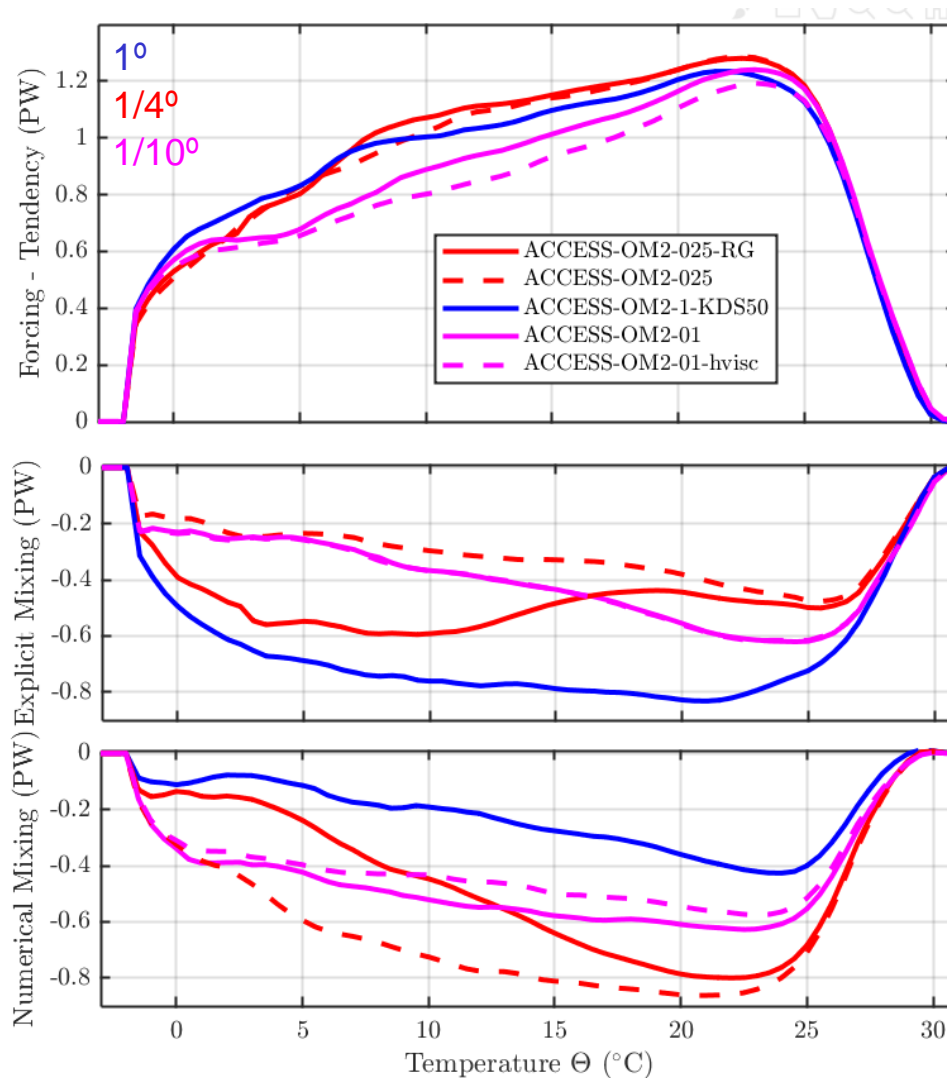
Adding along-isopycnal diffusion ($\sim 200\text{m}^2\text{s}^{-1}$) reduces numerical mixing by $\sim 60\%$. Diathermal budget suggests clean substitution.

However - there are significant mean state changes (Southern Ocean, Gulf Stream).



Note: Changes in vertical mixing negligible

Sensitivity to horizontal resolution and lateral viscosity



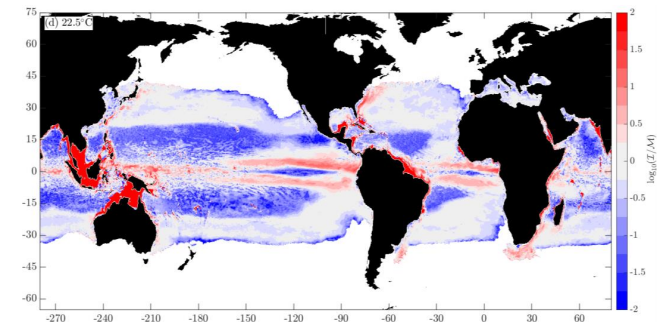
Generally, numerical mixing largest at 1/4 $^{\circ}$, followed by 1/10 $^{\circ}$ and finally 1 $^{\circ}$.

Partially because of **physical mixing differences across the resolutions** – choices of mixing coefficients made partially in anticipation of numerical mixing.

Increasing Smagorinsky biharmonic viscosity from 2- \rightarrow 3 in 1/10 $^{\circ}$ reduces numerical mixing by 10-15%.

Spectral analysis (coming soon) will yield more insight...

Summary/conclusions/questions



Key Conclusions:

- 1) Numerical mixing drives a large fraction of global warm-to-cold heat transport.
- 2) Sensitivity to resolution and physical mixing schemes suggests:
 - warm/tropical numerical mixing is largely diapycnal
 - Cold/boundary current numerical mixing has a large isopycnal component.

Open questions:

- 1) Does numerical mixing constitute a valid representation of absent physical mixing?
- 2) Are the simulations adversely affected (more work needed – spectra, biases)?

Questions and suggestions welcome!