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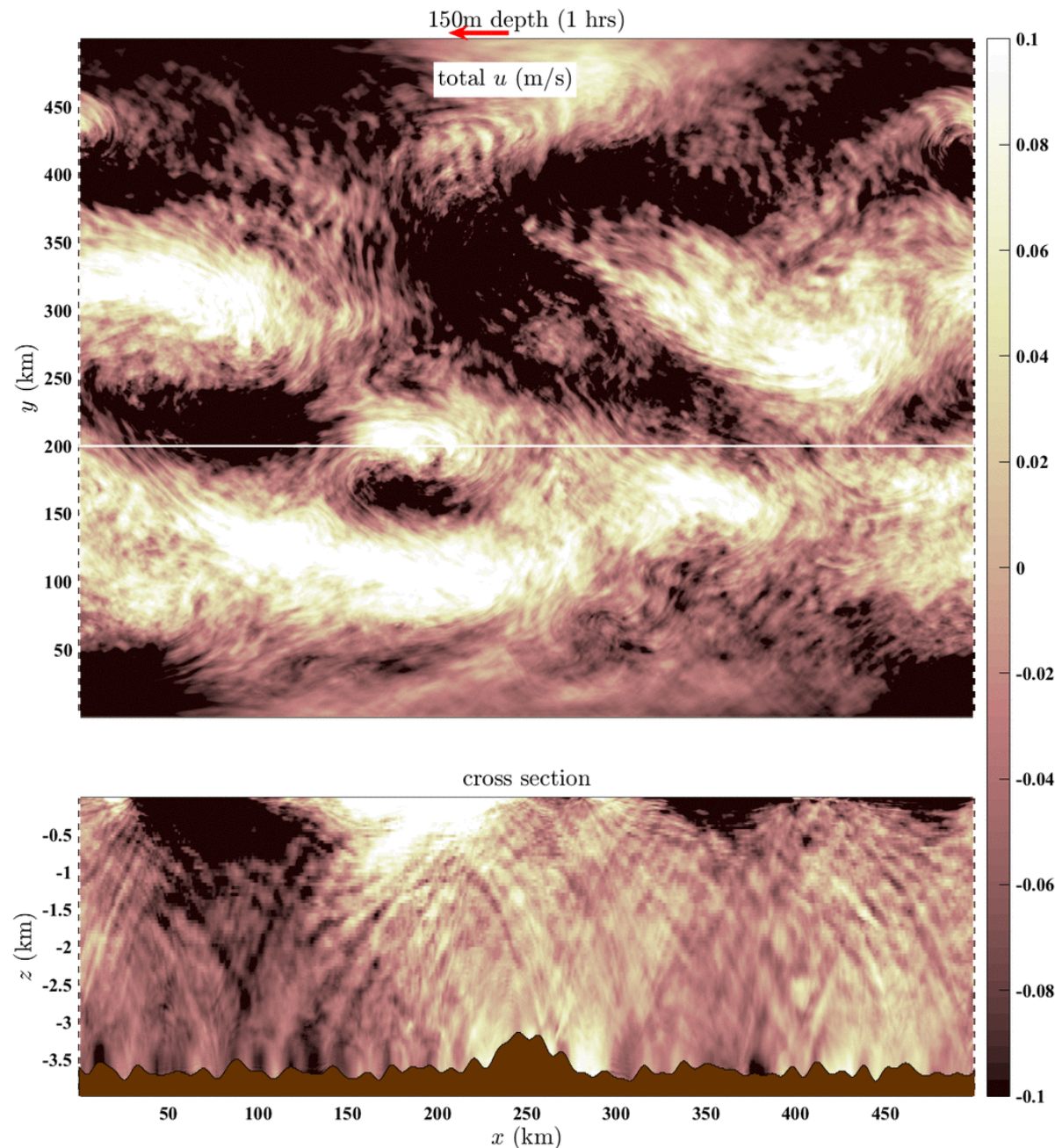


# Lagrangian Filtering

*A novel method for separating  
internal waves from non-wave  
flows in high-resolution  
simulations*

**COSIMA Virtual Workshop 2020**

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Gibson, Andy Hogg, Scott  
Bachman, Shane Keating, Nick  
Velzeboer**



# Why internal waves? A modelers perspective

- Internal waves are fast (hourly to daily) motions spanning scales from 200km to 100m
- They carry energy and momentum from the ocean boundaries into the interior
- They are responsible for both **mixing** (energy) and **forcing** (momentum) the ocean
- Many high-resolution models now resolve internal waves
- However, to quantify the wave energy and momentum in a model we need to be able to isolate internal waves from the rest of the flow...
- **Difficult! - spatial and temporal scales overlap with other phenomena**

**-> How do we uniquely define an internal wave in a model?**

# Defining an internal wave

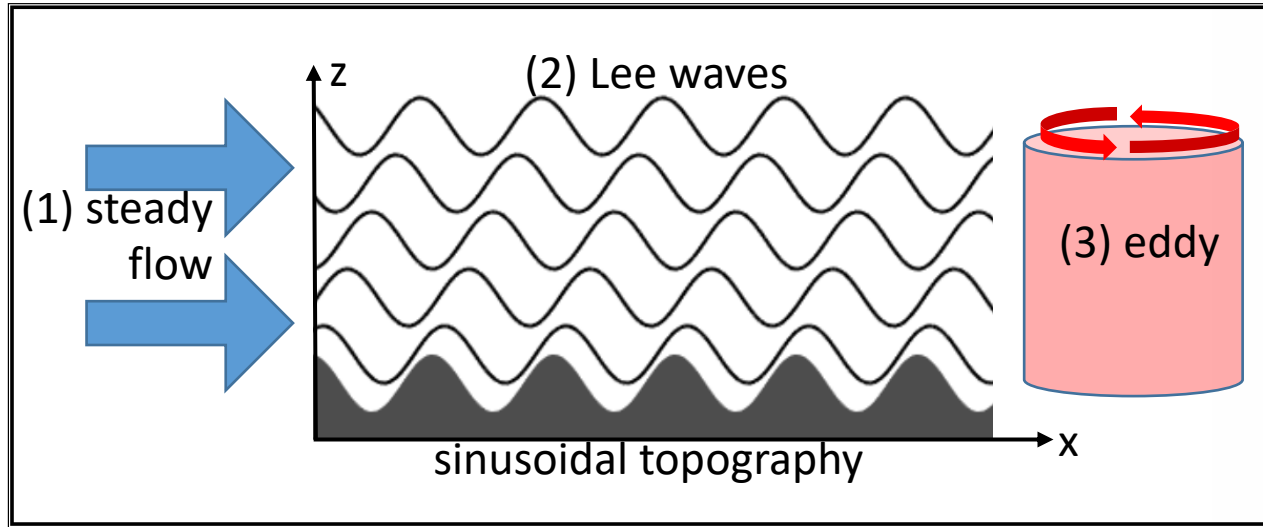
$$\omega^2 = \frac{f^2 m^2 + N^2 (k^2 + l^2)}{k^2 + l^2 + m^2} = f^2 + \frac{N^2 (k^2 + l^2)}{m^2} > f^2$$

- $\omega$  is the frequency,  $\vec{k} = (k, l, m)$  is the wavevector
- Internal waves have a minimum frequency = Coriolis parameter  $f$
- But they are also affected by any “mean” flows in which they propagate in two major ways
  - Effective vorticity:  $f^2 \rightarrow f_{eff}^2 = \left(f + \frac{\partial v}{\partial x}\right) \left(f - \frac{\partial u}{\partial y}\right)$
  - Doppler shifting:  $\omega \rightarrow \omega + (k u + l v)$
- A wave propagating with a mean flow will have a frequency less than  $f$  (and possibly zero; e.g. lee waves, other trapped waves).
- So there is no time-scale separation! This is a problem...

# So, what's the problem?

- Let's consider lee waves as an example

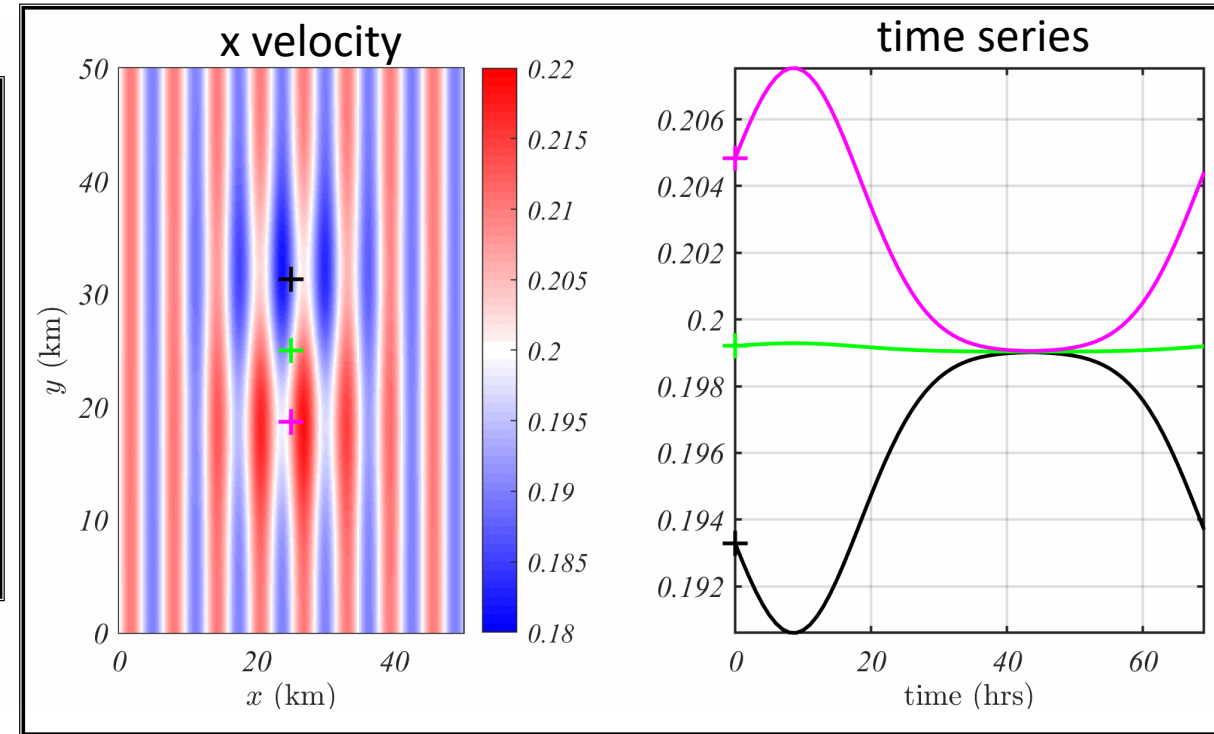
Schematic side-on view



When sampling at fixed points

- Trapped/lee wave signals are not observed in time series
- Eddy is seen as a fast motion due to advection past the fixed points
- Internal waves **cannot** in general be identified as “fast motions” when sampling at fixed points

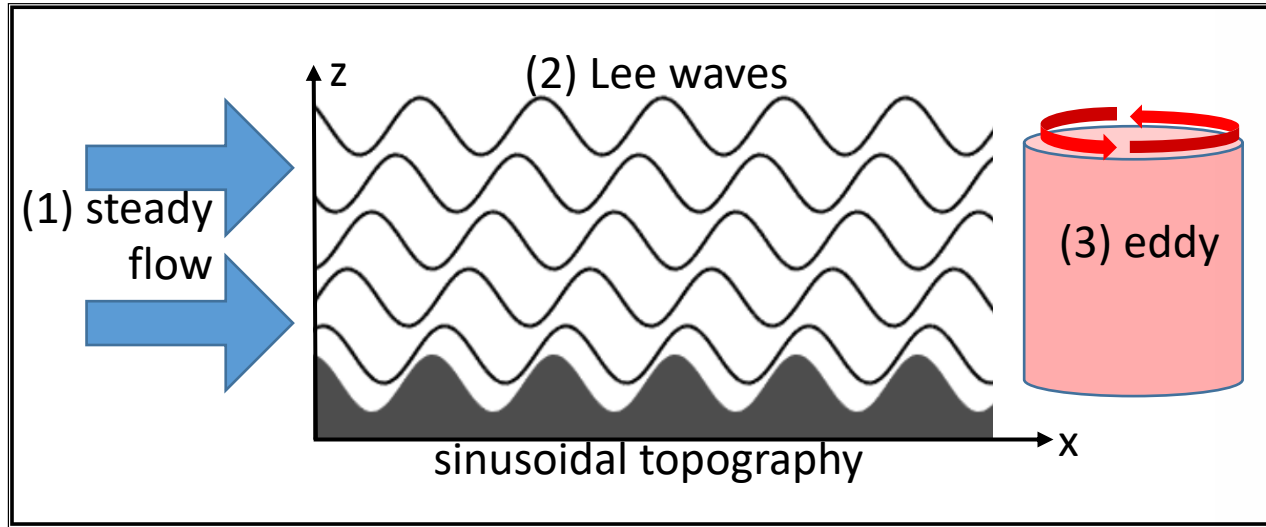
Top/plan view



# So, what's the problem?

- Let's consider lee waves as an example

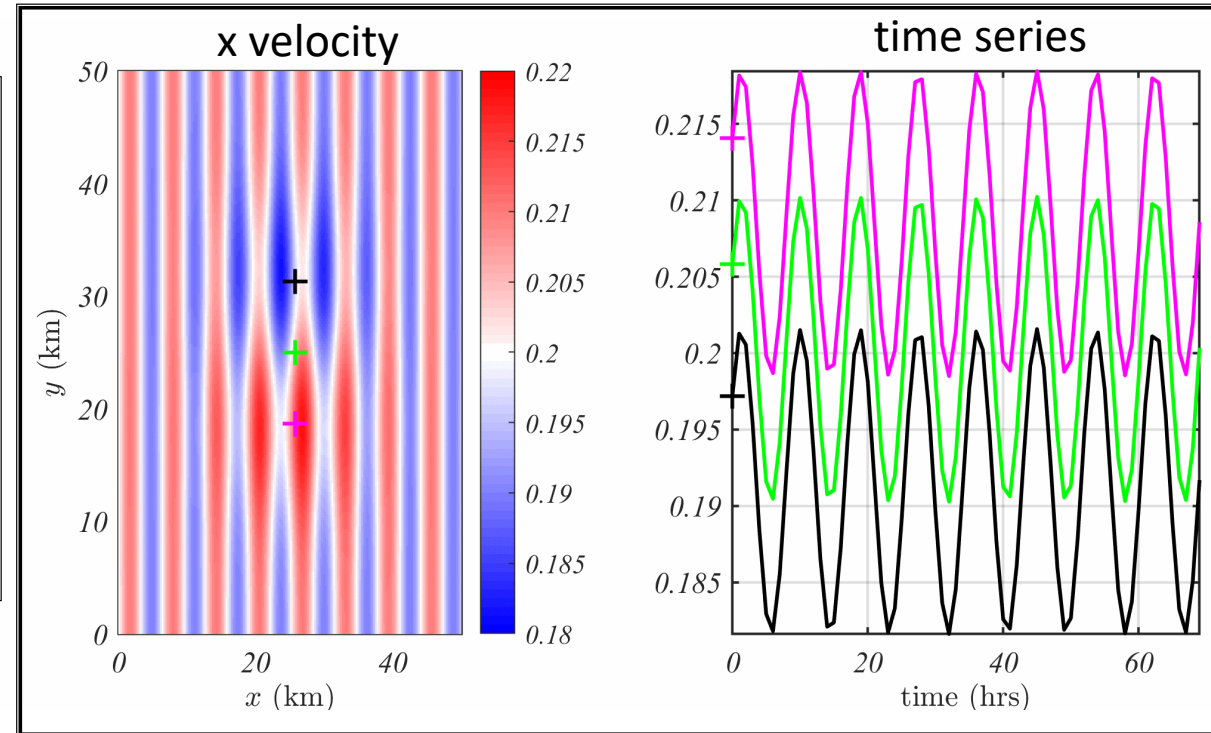
Schematic side-on view



When sampling at flow-following points

- Variation in space become a variation in time
- Trapped/lee wave signals are clearly observed
- Eddy is not observed since points move with the eddy.
- Internal waves **can** be identified as “fast motions” when sampling in a flow-following frame

Top/plan view



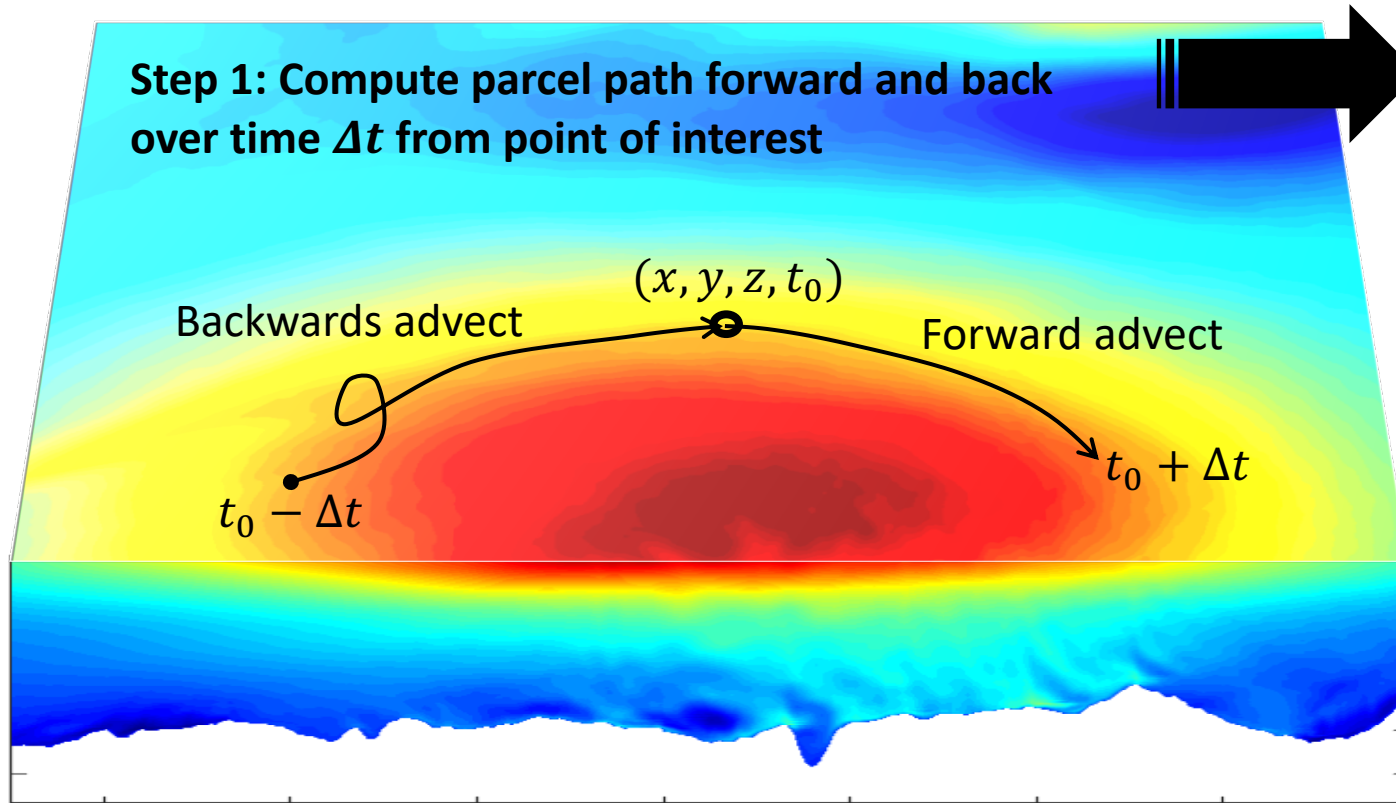


# Defining an internal wave

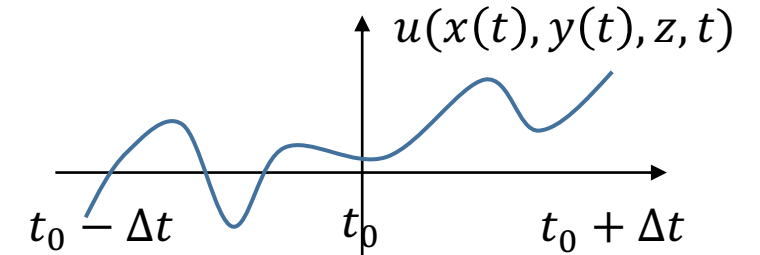
$$\omega^2 = f_{eff}^2 + \frac{N^2(k^2 + l^2)}{m^2} > f_{eff}^2 \quad \text{in a flow-following frame}$$

- The Doppler shift vanishes in a flow-following (Lagrangian) frame, since we are moving with the flow
- Internal waves therefore have a minimum frequency in such a frame
- There is (spatio-)temporal scale separation
- **We have the basis of a filter!**

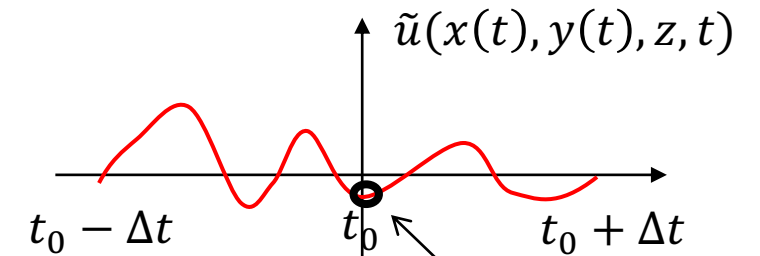
# Introducing the Lagrangian filter



**Step 2: Interpolate fields to parcel path**



**Step 3: Apply a high-pass filter**



**Step 4: Extract the value at  $t_0$**

**Step 5: Repeat for every point on the model grid, at every time of interest.**

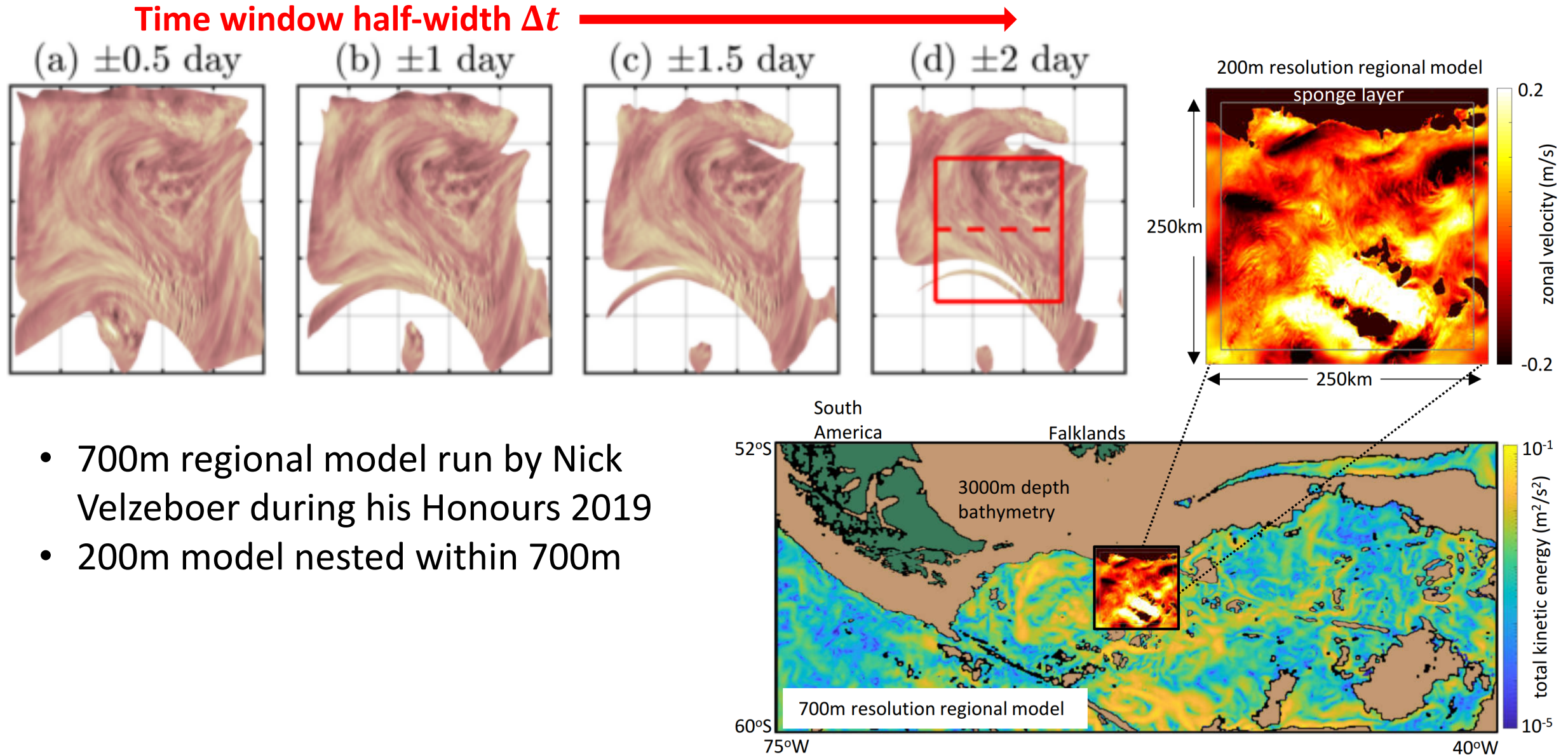
# Introducing the Lagrangian filter

## Parameter selections

1. **Filter cut-off frequency:** just below  $f$ , or  $f_{eff}$
2. **Time resolution of model output:** must be able to resolve the wave frequencies of interest. Typical periods are  $\sim 12$  hours  $\rightarrow$  hourly output
3. **Time window half-width  $\Delta t$ :** numerical choice. Smaller = less advection, so quicker.  $\Delta t = 1$  to 2 days satisfactory in most cases



# Example: Scotia Sea regional model

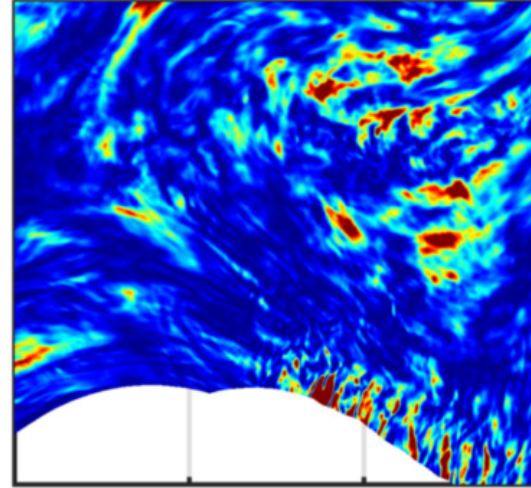


- 700m regional model run by Nick Velzeboer during his Honours 2019
- 200m model nested within 700m

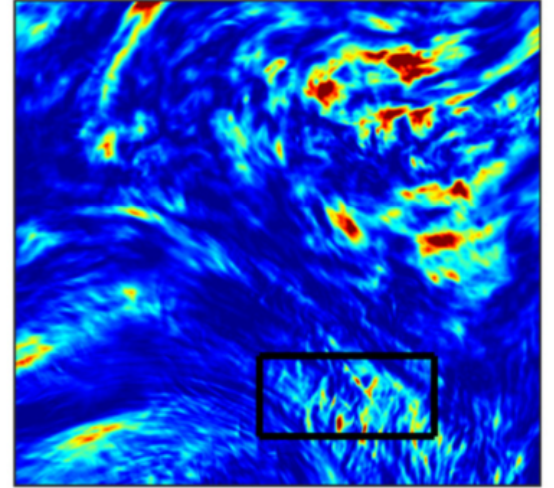
# Example: Scotia Sea regional model

- Comparison of Lagrangian filter with traditional fixed point (Eulerian) filter
- Eulerian misidentifies trapped waves as mean flow
- Underestimates wave energy by 20-50%

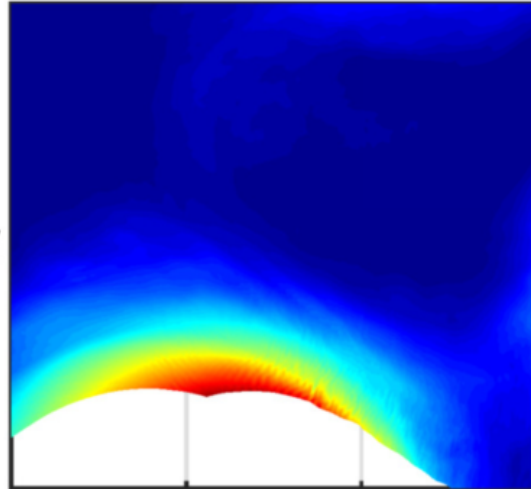
(a) Lagrangian wave



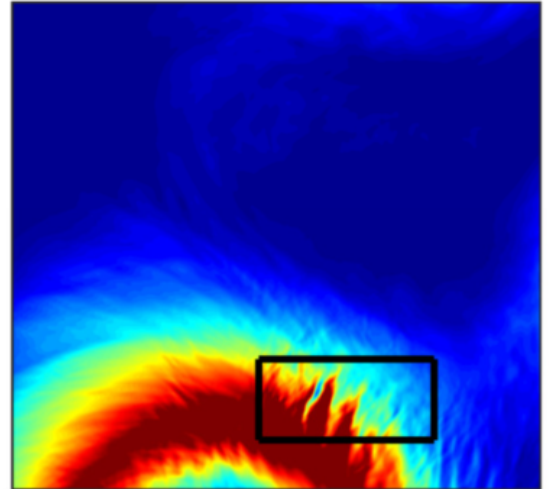
(b) Eulerian wave



(c) Lagrangian mean

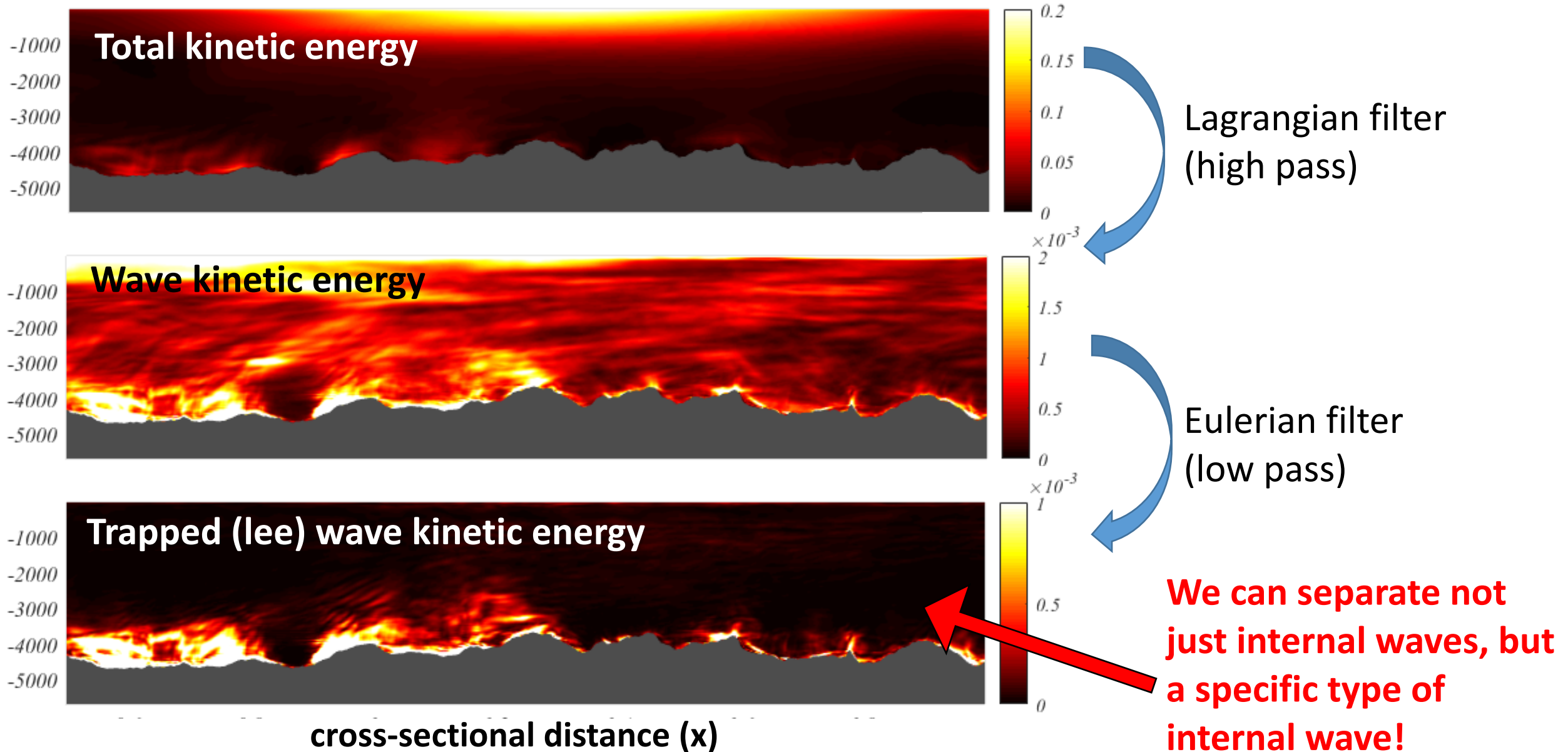


(d) Eulerian mean



# Example: Scotia Sea regional model

- We can apply filters sequentially to isolate specific phenomena...



# The Python package – now available

<https://github.com/angus-g/lagrangian-filtering>

- Available through *conda* package manager
- Advection kernel uses OceanParcels (<http://oceanparcels.org/>)
- Input: netcdf files for (u,v) at minimum, plus other variables of interest (e.g. T, S, rho, p)
- Output: netcdf files containing filtered wave fields on the same grid
- Can be trivially parallelised since every time and z-level is independent.
- Takes of order 5 min per time per level on a single core, or about 3 hrs for 1 month of hourly output parallelised over 500 cores on *gadi*



# Summary

- Novel method to uniquely separate internal waves in model flow fields
- Allows computation of wave energetics (see Jemima's talk)
- Now available as an easy-to-use Python package: <https://github.com/angus-g/lagrangian-filtering>. Feedback welcome!
- Methods paper in preparation for JAMES
- Example applications (thus far):
  - Nagai et al (2016) – Kuroshio model
  - Shakespeare and Hogg (2017,18,19) – spontaneous generation, and tidal forcing
  - Bachman et al (in rev.) – internal tides in a Coral Triangle regional model
- Features still under development; e.g.
  - Improving interpolation near topography
  - Vorticity-dependent filter cut-off frequencies
  - Support for arbitrary model grids
  - Diagnosis functions: e.g. extracting spectra along parcel paths